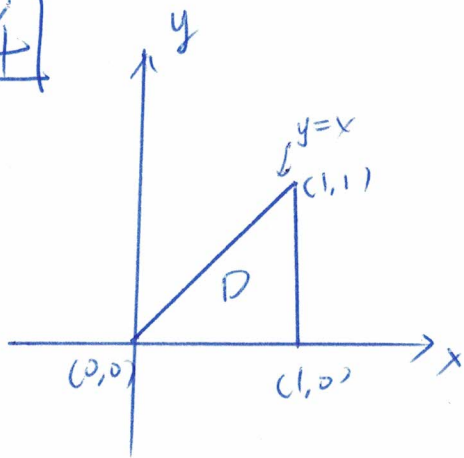
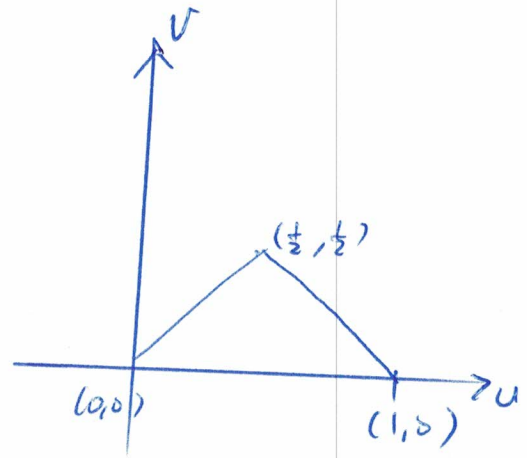


§ 6.2.

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\xrightarrow{T}



From problem, $x = u + v$ — ①

$y = u - v$ — ②

$$\textcircled{1} + \textcircled{2} : x + y = 2u \Rightarrow u = \frac{x+y}{2}$$

$$\textcircled{1} - \textcircled{2} : x - y = 2v \Rightarrow v = \frac{x-y}{2}$$

Thus, we have

$$T(x, y) = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix}$$

Three vertices of D are (0,0), (1,0), (1,1).

We put them into mapping T, then we get

$$T(0,0) = (0,0)$$

$$T(1,0) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

$$T(1,1) = (1,0)$$

So, we have integral region D^* is a triangle with vertices (0,0), $\left(\frac{1}{2}, \frac{1}{2}\right)$, (1,0).

By changing of variables,

$$\iint_D (x+y) \, dx \, dy = \int_0^1 \int_v^{1-v} [(u+v) + (u-v)] \left| \frac{\partial(x,y)}{\partial(u,v)} \right| \, du \, dv$$


= ...

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Let $A = \int_0^{\infty} e^{-4x^2} dx.$

$$\begin{aligned} A^2 &= \left(\int_0^{\infty} e^{-4x^2} dx \right) \left(\int_0^{\infty} e^{-4y^2} dy \right) \\ &= \int_0^{\infty} \int_0^{\infty} e^{-4x^2 - 4y^2} dx dy \end{aligned}$$

By polar coordinate, $x = r \cos \theta$, $y = r \sin \theta.$

From, , we know $0 \leq r < \infty$
 $0 \leq \theta \leq \frac{\pi}{2}.$

$$= \int_0^{\frac{\pi}{2}} \int_0^{\infty} e^{-4r^2} r dr d\theta$$

...