Preconditioning-based techniques for the convergence analysis of singularly perturbed convection-diffusion problems

Thái Anh Nhan [1]

Relja Vulanović [2]

We consider numerical methods for solving the linear singularly perturbed convection-diffusion problem,

$$\mathcal{L}u := -\epsilon u'' - b(x)u' + c(x)u = f(x),\ x \in (0,1),\ u(0) = u(1) = 0, \tag{1}$$

where ϵ is a small positive perturbation parameter, $0 < \epsilon \ll 1$. In general, the solution u has an exponential boundary layer near $x = 0$.

The problem (1) is discretized using finite-difference schemes on layer-adapted meshes. Generally speaking, such discretizations are not consistent uniformly in ϵ, so ϵ-uniform convergence cannot be proved by the classical approach based on ϵ-uniform stability and ϵ-uniform consistency. This is why previous proofs of convergence have introduced non-classical techniques (e.g., specially chosen barrier functions).

In this talk, we summarize our newly developed preconditioning-based approach—a suitable preconditioning of the discrete system is shown to yield a method that, uniformly in ϵ, is both consistent and stable. Using this technique, ϵ-uniform error bounds are obtained for the upwind [4, 3] and hybrid higher-order finite-difference schemes [2, 1].

References

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[1] Department of Mathematics and Science, Holy Names University, 3500 Mountain Blvd., Oakland, CA 94619, USA

[2] Department of Mathematical Sciences, Kent State University at Stark, 6000 Frank Ave NW, North Canton, OH 44720, USA