

# (IP) A Reynolds-robust preconditioner for the 3D stationary Navier–Stokes

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When approximating PDEs with the finite element method, large sparse linear systems must be solved. The ideal preconditioner yields convergence that is algorithmically optimal and parameter robust, i.e. the number of Krylov iterations required to solve the linear system to a given accuracy does not grow substantially as the mesh or problem parameters are changed.

Achieving this for the stationary Navier–Stokes has proven challenging: LU factorisation is Reynolds-robust but scales poorly with degree of freedom count, while Schur complement approximations such as PCD and LSC degrade as the Reynolds number is increased.

Building on the ideas of Schöberl, Benzi & Olshanskii, and Lee et al. [1, 2, 3], in this talk we present the first preconditioner for the Newton linearisation of the stationary Navier–Stokes equations in three dimensions that achieves both optimal complexity and Reynolds-robustness. The scheme combines augmented Lagrangian stabilisation to control the Schur complement, a divergence-capturing additive Schwarz relaxation method on each level, and a specialised prolongation operator involving non-overlapping local Stokes solves. The properties of the preconditioner are tailored to the divergence-free Scott–Vogelius discretisation [4].

We present 3D simulations with over one billion degrees of freedom with robust performance from Reynolds numbers 10 to 5000.

## References

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