Sept 15

$(Ax,y) = y^H (A x) = y^H A x =$

$= (y^H A) x$
$= (A^H y)^H x$
$= (x, A^H y)$

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orthogonal matrices preserve lengths

Q == orthogonal if $Q^H Q = I$

$\| Q x \|^2 = (Qx,Qx) = (x,Q^H Qx) = (x,x)$
$= \| x \|^2$

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limit of holder norms as $p \to \infty$

$\lim_{p\to\infty} [\sum |x_i|^p]^{1/p}$

assume that $\max |x_i| = |x_1|$

$[\sum |x_i|^p]^{1/p}$
$= [|x_1|^p + |x_2|^p +\ldots |x_n|^p]^{1/p}$
$= |x_1| [1 + |x_2/x_1|^p +\ldots |x_n/x_1|^p]^{1/p}$

$= |x_1| [k + \varepsilon]^{1/p}$

where k = number of x_i's s.t. $|x_i| = |x_1|$

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Proof of Cauchy - Schwarz

assume real inner products
$y \neq 0$

$0 \le (x - \beta y, x -\beta y)$
$= (x,x) -2 \beta (x,y) + \beta^2 (y,y)$
take
$\beta = (x,y)/(y,y)$
$\to$
$0 \le (x,x) - 2 (x,y)^2 /(y,y) + (x,y)^2 /(y,y)$
$0 \le (x,x) - (x,y)^2 /(y,y)$
$\to$
$(x,y)^2 \le (x,x) (y,y)$

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$|| x ||_2^2 = \sum x_i^2$

$|| x ||_1^2 = [\sum |x_i|]^2$

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$\sum |x_i| = \sum x_i \operatorname{sign}(x_i) \le ||x||_2 || s ||_2$
cauchy-schwarz $\to$
s = vector of ± 1

$||s|| = \sqrt{n}$

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