PRINCIPLES OF PARALLEL ALGORITHM DESIGN

- **Dependency graphs. Parallelism in algorithms**
- **Characteristics on tasks and interactions.**
- **Decomposition methods**
- **Mapping and load balancing**

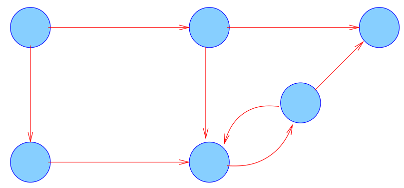
Tasks and Communications

➤ Designing parallel programs is all about tasks [which will run in parallel]

➤ Tasks can have fine or coarse granularity

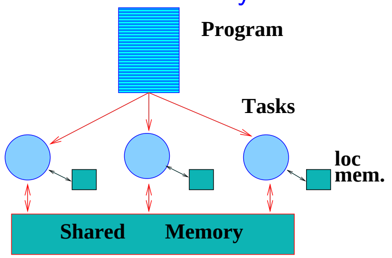
➤ Tasks can communicate with each other ...

... either explicitly as in message passing, or through memory access + some sychronization mechanism - as in shared memory models



The two prevailing programming viewpoints

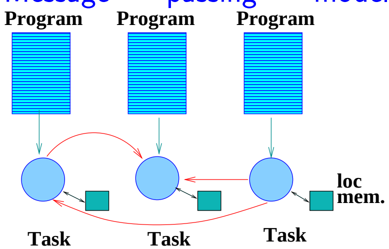
Shared memory model



➤ Single program forks tasks –

➤ Open MP, threads

Message passing model



➤ Usually Single Program Multiple Data

➤ MPI, PVM, ..

➤ Important: in both cases, a standard language (e.g., FORTRAN, C, C++) is augmented by a either communication library (MPI) or directives (openMP).

Designing parallel programs

There are four main steps:

1. Decomposition into tasks

2. Identify Communication patterns (local, global, broadcast, ...)

3. Agglomeration and load balancing (grouping tasks together to improve performance, simplify coding, achieve balanced work among PEs, ...

4. Mapping: process of assigning tasks to processors.

[See Ian Foster "Designing and building parallel programs"]

Tasks and dependency graphs

➤ First issue: identify "tasks" considered as indivisible units of work. "Fine-grain" parallelism: tasks are grouped in very small units, e.g., at the level of artithmetic operations. "Coarse grain" parallelism: tasks are bigger.

➤ Tasks depend on each other : the result of one task may be required by another task.

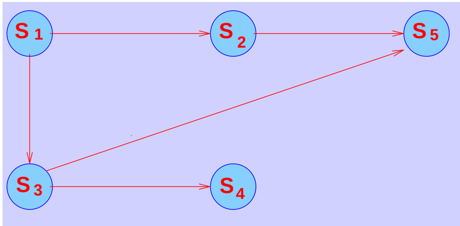
➤ Important tool: Dependency Graph, which sets the dependencies. There is an (oriented) edge from task A to task B if B cannot be started before A is completed.

A simple example

➤ Consider the following tasks

S1: $A = B + C$	First task
S2: $B = B * A$	S1 must be done first
S3: $A = A + 1.0$	S1 must be done first
S4: $C = A * 2 + 3$	S3 must be done first
S5: $X = A + B$	S3, S2 must be done first

➤ Dependence graph :
Possible schedule:
(1) S1;
(2) S2, S3;
(3) S4, S5.

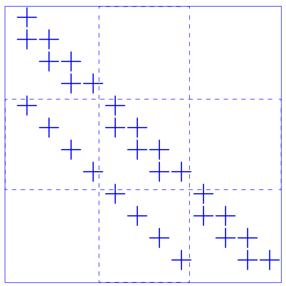


Example: triangular system solutions

Example: Solve the linear system of equations

$$Ax = b$$

where A has the following structure:

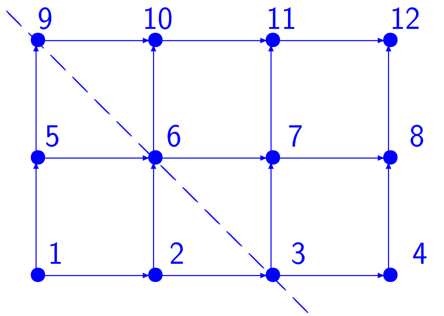


➤ 12 tasks altogether [one for solving each equation]

➤ Task number i: solve equation number i.

➤ Dependency: unknown i requires previous unknowns to which it is coupled by an equation. Thus, x_6 requires unknowns x_5 and x_2.

➤ Dependency graph :



Sets of equations that can be solved in parallel:

$$\{1\}, \{2, 5\}, \{3, 6, 9\}, \{4, 7, 10\}; \{8, 11\}; \{12\}$$

Some definitions

- Degree of concurrency = max number of tasks which can be performed in parallel
- Critical path = longest (directed) path between any two nodes in the graph == Longest path from a start node (no incoming edges) to a finish node (no outgoing edges). Path not unique.
- Nodes can be weighted to include a cost measure in case the tasks have different costs
- Weighted path length = the sum of the weights of the nodes along the path. "Critical path length" = maximum path length. If weights represent time, the algorithm cannot be executed in less time than the critical path length.

- Mapping: the mechanism by which tasks are assigned to processors. May also refer to method by which Data is assigned to processors.

Example: For the example of the sparse triangular solve, one can define the weights on the nodes as the number of operations performed to compute x_i. Observe that at each node this cost is

$$w_i = 1 + 2 * (\# \text{ incoming edges})$$

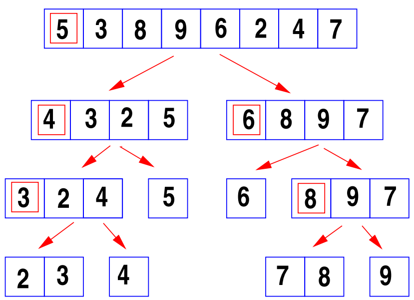
✍1 What is the critical path length in this case?

Decomposition

- Decomposition refers to the process of splitting the initial computation into independent tasks.

Recursive decomposition: Divide and conquer paradigm

Example: Recursive Quicksort [sorting will be covered in detail later]



Example: Search a data item in an array.

Data decomposition: Concurrency is unraveled by primarily splitting data. Best examples: Adding n numbers, Adding 2 matrices, product of matrices, ..

Example: Go back to the sum of n numbers

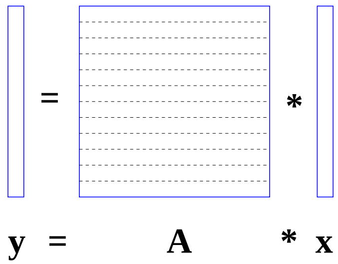
$S = x_1 + x_2 \cdots + x_n = S_1 + S_2$ where

$S_1 = x_1 + x_2 \cdots + x_m$

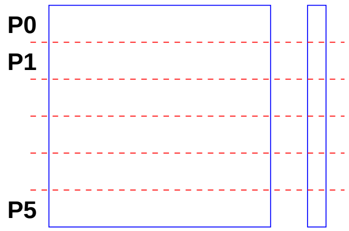
$S_1 = x_{m+1} + x_{m+2} \cdots + x_n$

Example: Matrix-vector product

➤ To compute: $y = Ax$ where A is $m \times n$, $x \in \mathbb{R}^n$.
➤ Consider row-based algorithm: $y_i = A(i,:).x$
➤ y_i = Dot product of row i with x
➤ Can execute these tasks in parallel



➤ Can also divide the rows into blocks.
➤ We say that we have defined a tasks with ‘coarser’ granularity.
➤ The result now is a block of y instead of just one component



In summary: Major types of decomposition

Data Decomposition Problem divided based on splitting data

Recursive Decompositon Divide and conquer - e.g. Quicksort

Exploratory decomposition: Is usually related to problems which involve a search space (best example: game of chess). Different search spaces are explored in parallel by different processors

Speculative decomposition: In case of branches in the algorithm, different “speculative” branches can be taken by different processors

Mapping and load balancing

One of the biggest sources of loss of efficiency is a poor “load balancing”, where some processors are idle while others are busy.

Two two ways to achieve load-balancing:

1. Static mapping: estimate various task and communication costs and schedule tasks statically. Tasks / data are distributed in a certain way at the start (no changes during execution).

2. Dynamic mapping: Tasks / data are distributed/ redistributed during execution.

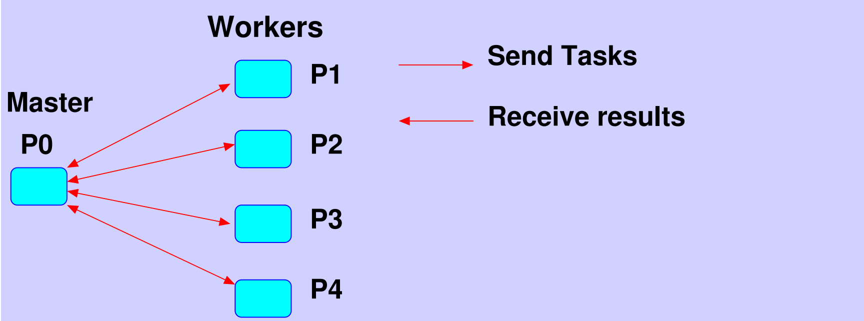
➤ Static mapping schemes: typically achieved by *mapping the data*.

➤ Many such examples related to matrix computations. Another example: Domain decomposition by graph partitioning.

➤ Dynamic scheduling schemes: best example is the ...

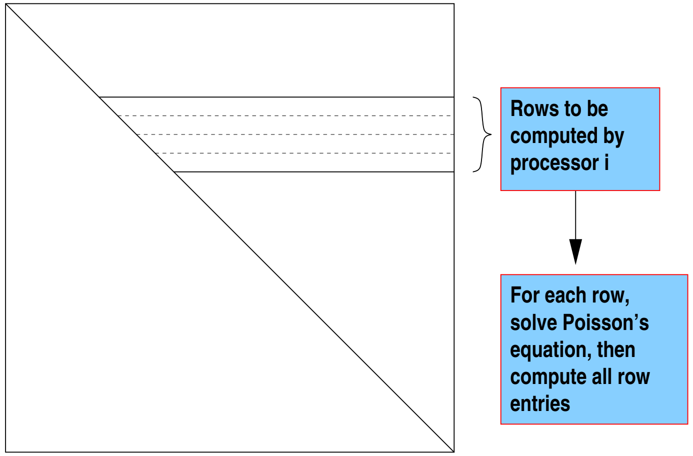
Master-Worker model. A master processor determines where each task is to be performed. May send off tasks to other processors or may give pointers as to what to do (see next example). May collect results back - or may probe processors to make decisions for next steps. Main point: all decisions, in particular which task performed where, are determined by the Master

The Master-Worker model

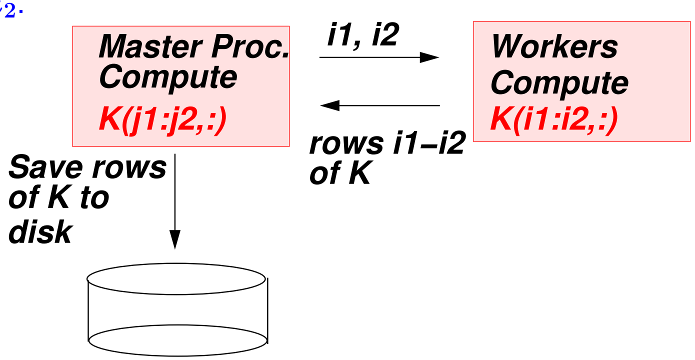


Example: A real-life example from Materials Science. Problem is to compute a dense symmetric matrix. Each row is *very* expensive to compute.

➤ Only lower portion of K matrix saved [incomplete rows]



➤ Master-worker model: A master sends indices i_1, i_2 of a block to be computed. Many factors are considered to determine the block $i_1 - i_2$.



Task pool model

➤ Processors take tasks from a global pool (again "global view" but tasks may be distributed). The pool itself can be dynamic.

➤ Can be entirely decentralized - (distributed decisions) or centralized (book-keeping of tasks is done in one location).

➤ Tasks are essentially moved around – so one requirement: little data to be moved along.