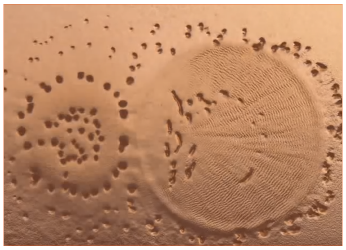
REU: Complex Systems and Pattern Formation

Summer 2018: June 18 – July 27

Pattern formation is the study of mechanisms that lead to the appearance of simple or complex spatial-temporal patterns. It is motivated in part by the observation of strikingly similar patterns in apparently unrelated physical systems. In this REU, participants will conduct mathematical research in the area of pattern formation from a viewpoint of dynamical systems and differential equations, using both analytical and computational tools.



Self-organized Clusters in Diffusive Run-and-Tumble Processes

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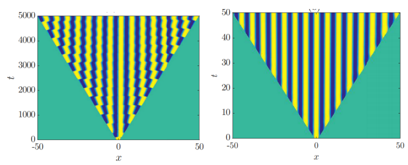
Abstract

We analyze a simplistic model for run-and-tumble dynamics, motivated by observations of complex spatio-temporal patterns in colonies of myxobacteria. In our model, agents run with fixed speed either left or right, and agents turn with a density-dependent nonlinear turning rate, in addition to diffusive Brownian motion. We show how a very simple nonlinearity in the turning rate can mediate the formation of self-organized stationary clusters and fronts. Phenomenologically, we demonstrate the formation of barriers, where high concentrations of agents at the boundary of a cluster, moving towards the center of a cluster, prevent the agents caught in the cluster from escaping. Mathematically, we analyze stationary solutions in a four-dimensional ODE with a conserved quantity and a reversibility symmetry, using a combination of bifurcation methods, geometric arguments, and numerical continuation. We also present numerical results on the temporal stability of the solutions found here.



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Universal wave-number selection laws in apical growth

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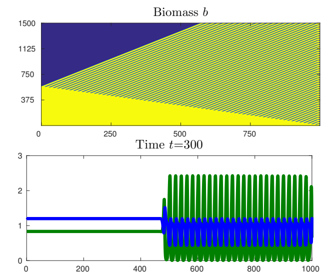
We study pattern-forming dissipative systems in growing domains. We characterize classes of boundary conditions that allow for defect-free growth and derive universal scaling laws for the wave number in the bulk of the domain. Scalings are based on a description of striped patterns in semibounded domains via strain-displacement relations. We compare predictions with direct simulations in the Swift-Hohenberg, the complex Ginzburg-Landau, the Cahn-Hilliard, and reaction-diffusion equations.



Advection and autocatalysis as organizing principles for banded vegetation patterns

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Abstract

We motivate and analyze a simple model for the formation of banded vegetation patterns. The model incorporates a minimal number of ingredients for vegetation growth in semi-arid landscapes. It allows for comprehensive analysis and sheds new light onto phenomena such as the migration of vegetation bands, their alignment with contour lines, and the interplay between their upper and lower edges.

Program Description:

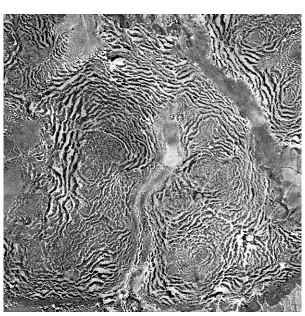
Six weeks (June 18 – July 27, 2018) on the U of Minnesota campus in Minneapolis.

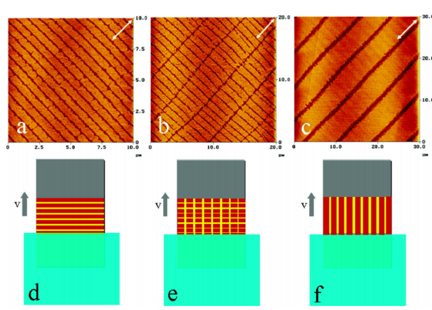
Use analysis and numerical simulations to gain insight into dynamics of patterns, in particular the selection and orientation of banded patterns, with possible applications to vegetation banding, recurrent precipitation, or Langmuir-Blodgett transfer.

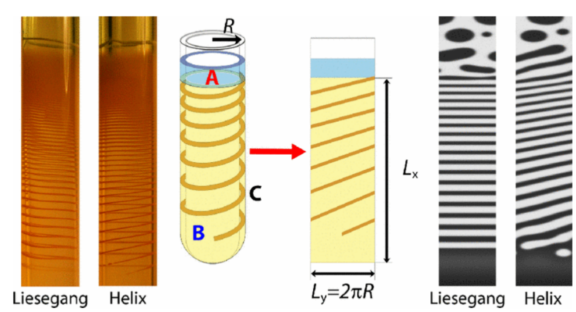
Participants will receive a stipend of $3,000 and up to $1,000 for travel, room, and board.

Successful Applicants:

- Are motivated undergraduate students
- Need no prior research experience
- Should have had a course in differential equations or dynamical systems
- May have higher-level coursework
- May have familiarity with or an interest in learning Mathematica or Matlab
- Must be US citizens or permanent residents
- Must not complete their undergraduate degree before summer 2018







Contact: Arnd Scheel (scheel@umn.edu). Program sponsored by the NSF.

Application Deadline: Febrary 16, 2018. For more information and **to apply**, go to:

`http://www.math.umn.edu/~scheel/reu/reu-opportunities.html`