On Renormalons in Supersymmetric Field Theories and Comments on Resurgence

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Resurgence and Transseries in Quantum, Gauge and String Theories
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I. Introduction

☛ Resurgence and trans-series, a breakthrough in "constructive" mathematics, ~ 1980s (G. Edgar, ArXiV 0801.4877)

✷ Perfectly works in QM and (some) weakly coupled FT

✎ Why it does not work in strongly coupled FT ?

✷ A (messy) analog is OPE (1960s-70s)

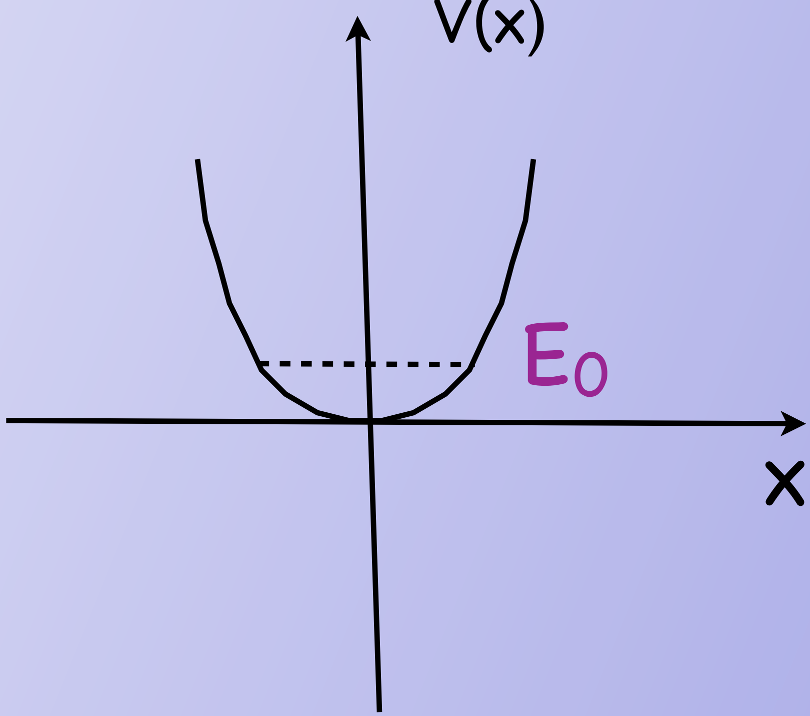
✷ ✷ ✷ II. General remarks;
III. Renormalons in SUSY

Zinn-Justin, Berry, **Jentschura, Dunne, Beneke's talks help!**

$$f(g^2) = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{k-1} a_{n,k,l}\, g^{2n} \left[\exp\left(-\frac{S}{g^2}\right)\right]^k \left[\log\left(-\frac{1}{g^2}\right)\right]^l$$

Quantum Mechanics +

$H = p^2/2 + (\omega^2/2)\, x^2 + g^2 x^4$



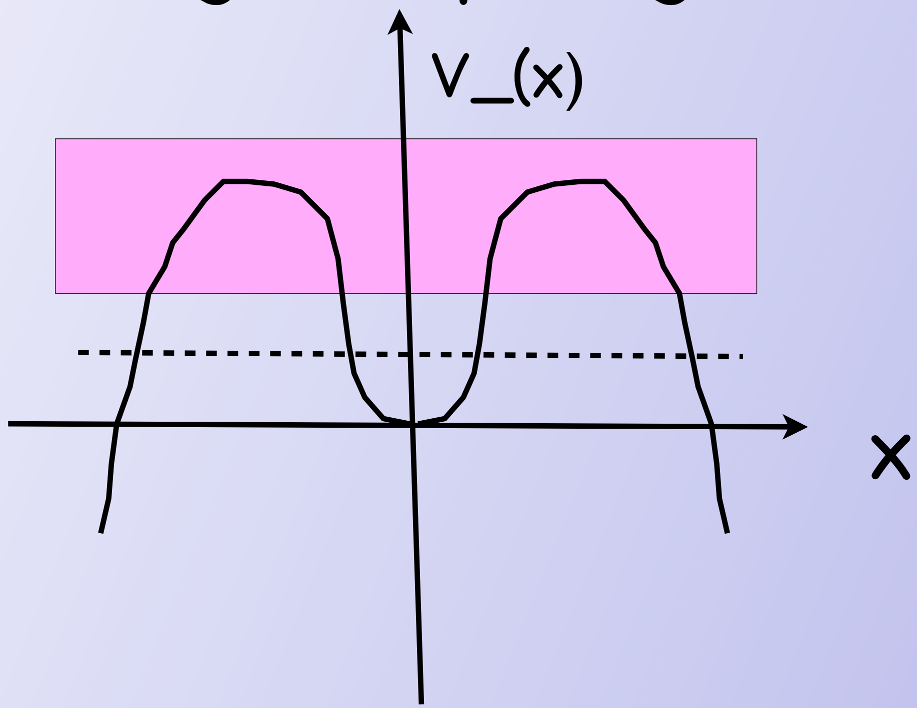
$E_0 = (\omega/2)\, \Sigma_n c_n g^{2n}$

$$c_n \sim (-1)^{n+1}\frac{3^n\sqrt{6}}{\pi^{3/2}}\,\Gamma(n+\tfrac{1}{2})$$

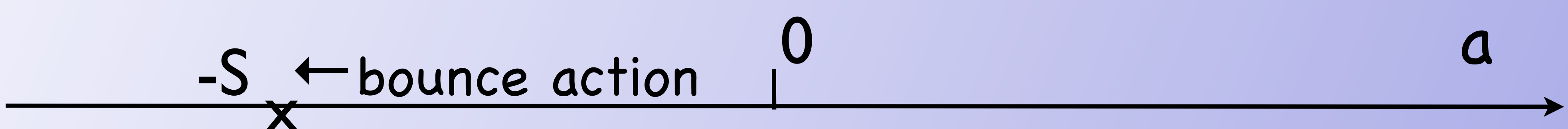
Dyson argument (1950s), Vainshtein (1964)

a) $g^2 \rightarrow - g^2$,

b) Im $E_0 = (\Sigma_n c'_n g^{2n})\exp(-S/g^2)$



Borel summable: $BE_0(a) = (\omega/2) f(a) = (\omega/2)\ \Sigma_n c_n g^{2n}/n!$

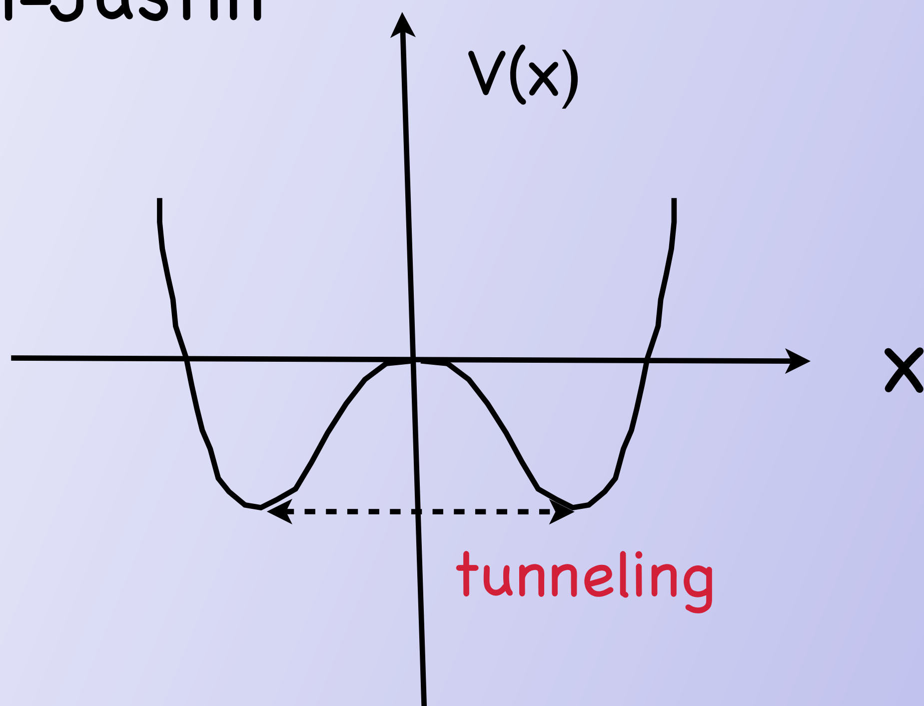


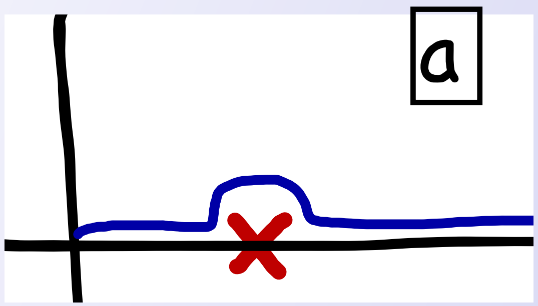
$$LBE_0(g^2) = (\omega/2) \int_0^\infty da\, \exp(-a/g^2)\, f(a)$$

Expanding $f(a)$ in $LBE_0(g^2)$ we get g-series term by term

Non-Borel-summable example: double well potential:

Bogomolny, Zinn-Justin





$$LBE_0(g) = (\omega/\sqrt{2})(\Sigma'_n c_n g^{2n} + e^{-S_{inst}} \Sigma'_n c_n' g^{2n} + \ldots)$$

II. AF Field Theory

In AF strongly coupled field theory this program can be carried out only if the theory is

✷ is exactly solvable;

✷ ✷ can be treated (perhaps, after a deformation) quasiclassically.

Examples of field theory

4D Yang-Mills or QCD:

$$L = -(1/4g^2)\, G_{\mu\nu}^{a}\, G^{\mu\nu a} \;+\; \boxed{i\bar{\psi}\not{D}\psi}$$

or

2D CP(N-1) model:

$$L = G_{A\bar{a}}\, \partial_\mu \Phi^{\bar{A}}\, \partial^\mu \Phi^a$$

$$G_{A\bar{a}} = \frac{2}{g^2} (\partial/\partial\bar{\Phi}^A)\ \partial/\partial\Phi^a \log \Sigma_{A=a} (1+\bar{\Phi}^A\ \Phi^a)$$

Problem: What you see in Lagrangian is NOT the asymptotic state which could be detected. Neither is g^2 ☹

Coupling constant g^2 is NO longer constant

Through dimensional transmutation

dynamical scale

$$g^2(Q) = \frac{S_0}{\beta_0 \log (Q/\Lambda)}$$

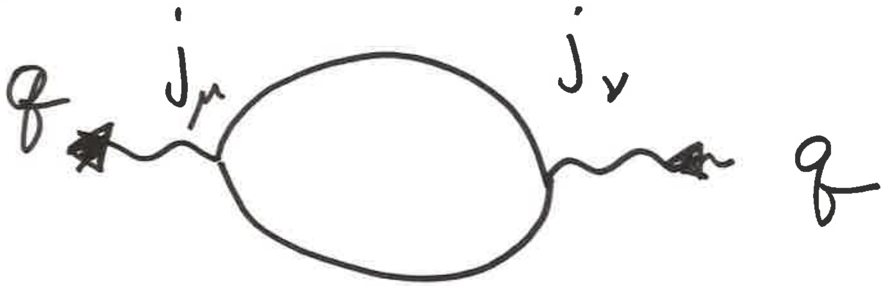
1st coeff.

$S_0 = 8\pi^2$ YM
$= 4\pi$ CP

$$\exp[-S_0/(\beta_0\, g^2(Q)) = \Lambda/Q$$

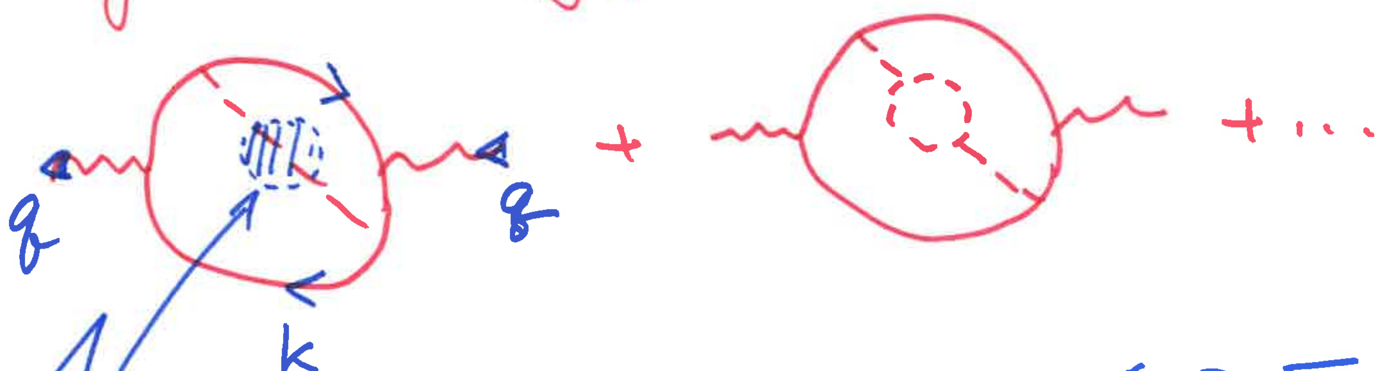
If $Q \sim \Lambda$ then g^2 becomes undefined!!!

$$Q^2 \equiv -q^2$$



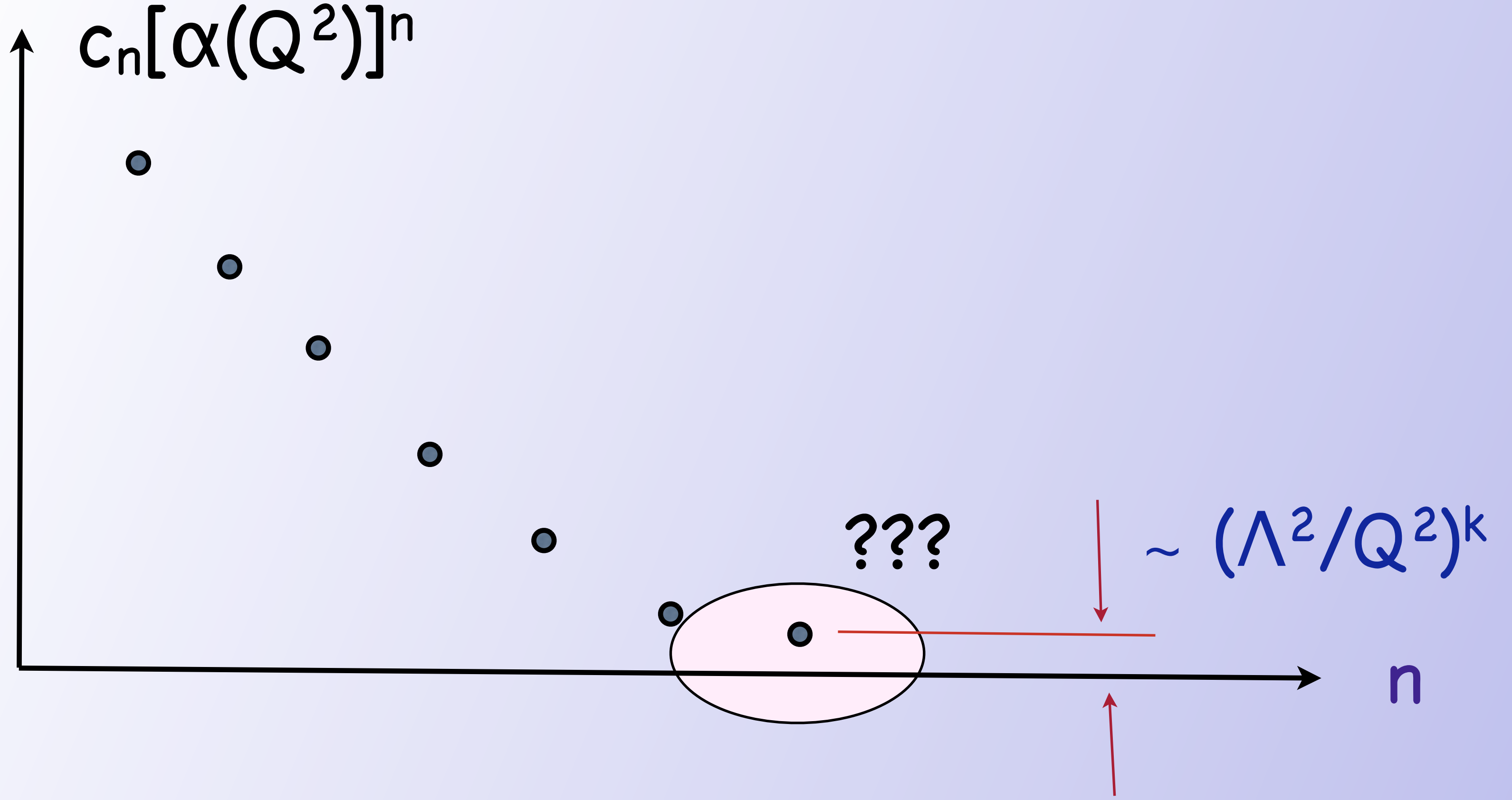
$$D(Q^2) \sim \# \left\{ 1 + c_1 g^2(Q^2) + c_2 (g^2)^2 + \ldots \right\}$$

Feynman diagrams:



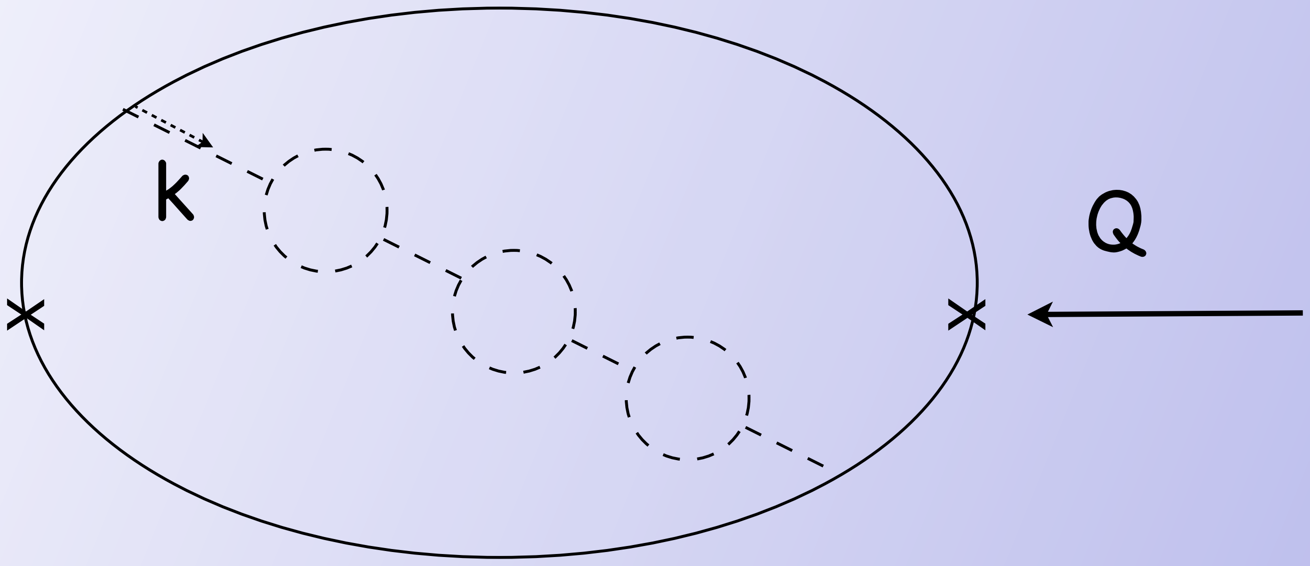
soft ?

$$\mathcal{D}_{\mu\nu} \neq g_{\mu\nu} \frac{1}{k^2} \Rightarrow \text{OPE} \leftrightarrow \text{Renormalons}$$



Superasymptotics

$$-\frac{2}{3}N_f \to \beta_0$$



$$D(Q^2) = \frac{1}{Q^4}\,\alpha_s \sum_{n=0}^{\infty}\left(\frac{\beta_0\alpha_s}{4\pi}\right)^n \int dk^2\, k^2 \left(\ln\frac{Q^2}{k^2}\right)^n , \qquad \alpha_s \equiv \alpha_s(Q^2)$$

$$D(Q^2) = \frac{\alpha_s}{2}\sum_{n=0}^{\infty}\left(\frac{\beta_0\alpha_s}{8\pi}\right)^n \int dy\, y^n\, e^{-y}, \qquad y = 2\ln\frac{Q^2}{k^2}$$

$y \sim n$ or $k^2 \sim Q^2 \exp\left(-\frac{n}{2}\right)$	$n_* = 2\ln\frac{Q^2}{\Lambda^2}$

Master formula (simplified)

$$D(Q) = \sum C_{0,n}(\not{\lambda}) \left(\frac{1}{\ln Q^2/\Lambda^2}\right)^n$$

$$+ \sum C_{1,n}(\not{\lambda}) \left(\frac{1}{\ln Q^2/\Lambda^2}\right)^n \left(\frac{\Lambda}{Q}\right)^{d_1}$$

$$+ \sum C_{2,n} \left(\frac{1}{\ln Q^2/\Lambda^2}\right)^n \left(\frac{\Lambda}{Q}\right)^{d_2} + \ldots$$

Omitted:
- $\log_m = \log \log \ldots \log Q^2/\Lambda^2 \longleftrightarrow \beta_{2,3,\ldots} \neq 0$
- Anomalous dimensions $\left(\log Q^2/\Lambda^2\right)^{\gamma_i}$

Wilson 69, 73
SVZ, 1970s

What can we do In practice ? → OPE

We have to introduce a separation (auxiliary ~~arbitrary~~) scale $\boxed{\mu}$

Cf. K from Jentschura's talk

$$C_{jn} \to C_{jn}(Q, \mu)$$

$$\left(\frac{\Lambda}{Q}\right)^{d_e} \to \left(\frac{\Lambda}{Q}\right)^{d_e} f(\mu/\Lambda)$$

Auxiliary μ cancels in the Master formula

$\mu \sim (a\ \#) \times \Lambda$

Examples

$* \; CP(N-1)\,, \quad \boxed{N\to\infty}$

$$i\int \langle T_\mu^\mu(x)\,,\, T_\alpha^\alpha(0)\rangle\, e^{iqx}\, d^2x$$

SUSY

convergent

$$\Pi = \sum_k C_k \left(\frac{\Lambda}{Q}\right)^{2k} = \sum{}' C_k\, e^{-4\pi k/g^2(Q)}$$

No logarithms

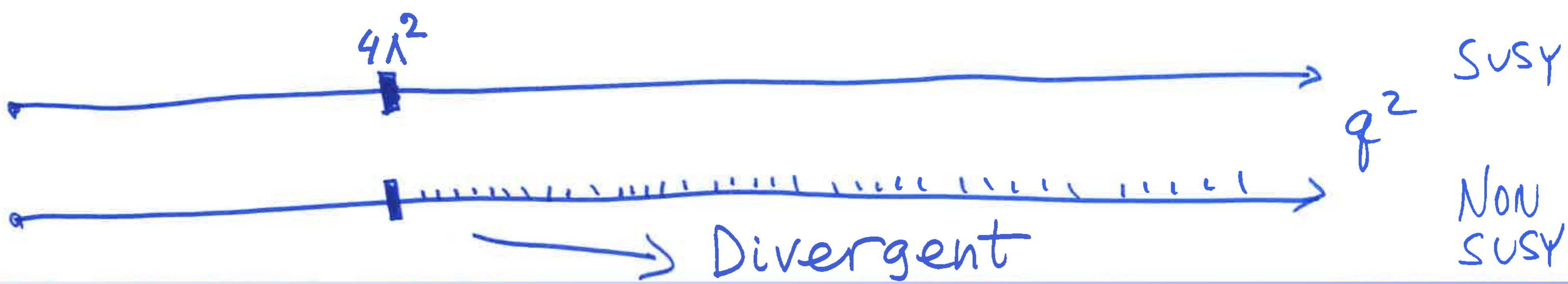
No RENORMALONS

* Singularities only at positive real q^2.

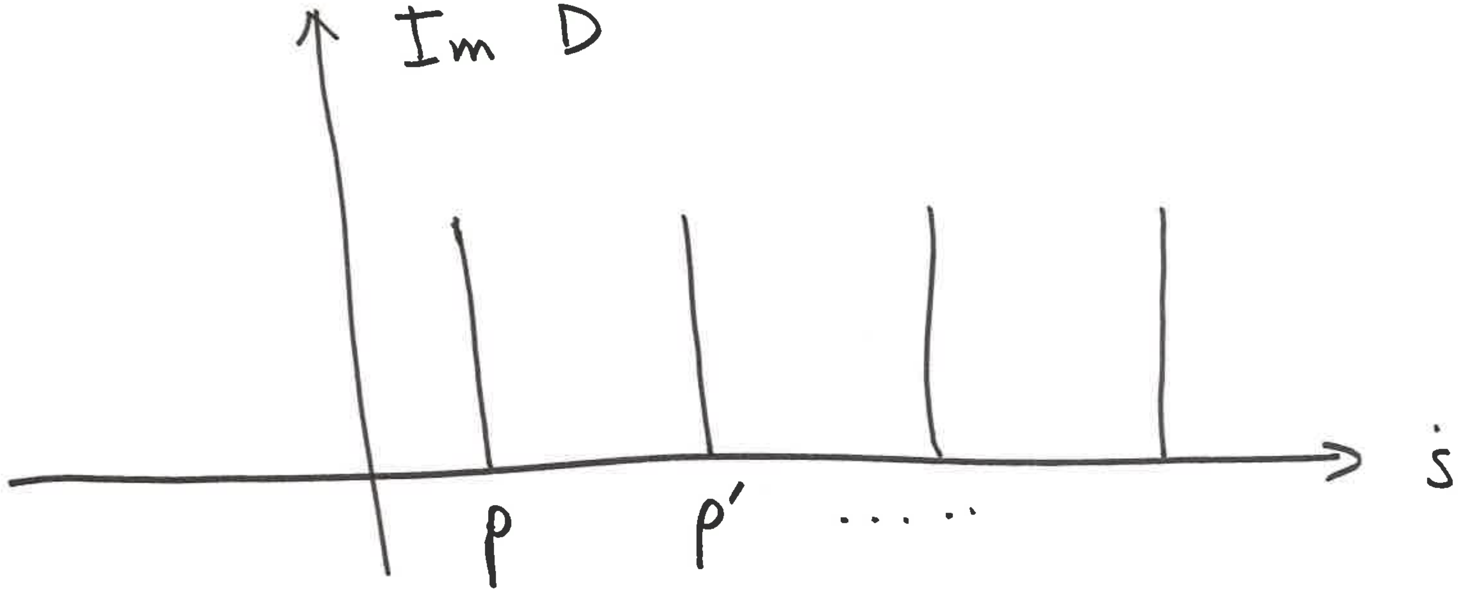
* No singularities ~~oth~~ elsewhere in the complex plane of Q^2.

't Hooft 1970s

CP(N-1) NON-SUSY vs SUSY



Example 2: Comb of resonances (Large N QCD)



$$D = -\frac{N}{12\pi^2}\,\psi(z) + \text{const.} \qquad \propto \sum_{k=0}^{\infty} \frac{1}{z + k + 1}$$

$$z = \frac{Q^2 + m_\rho^2}{3\, m_\rho^2}$$

appr

$$\psi(z) \sim \log z - \frac{1}{2z} - \sum_{n=1}^{\infty} \frac{B_{2n}}{2n}\,\frac{1}{z^{2n}}$$

⟵ divergent factor

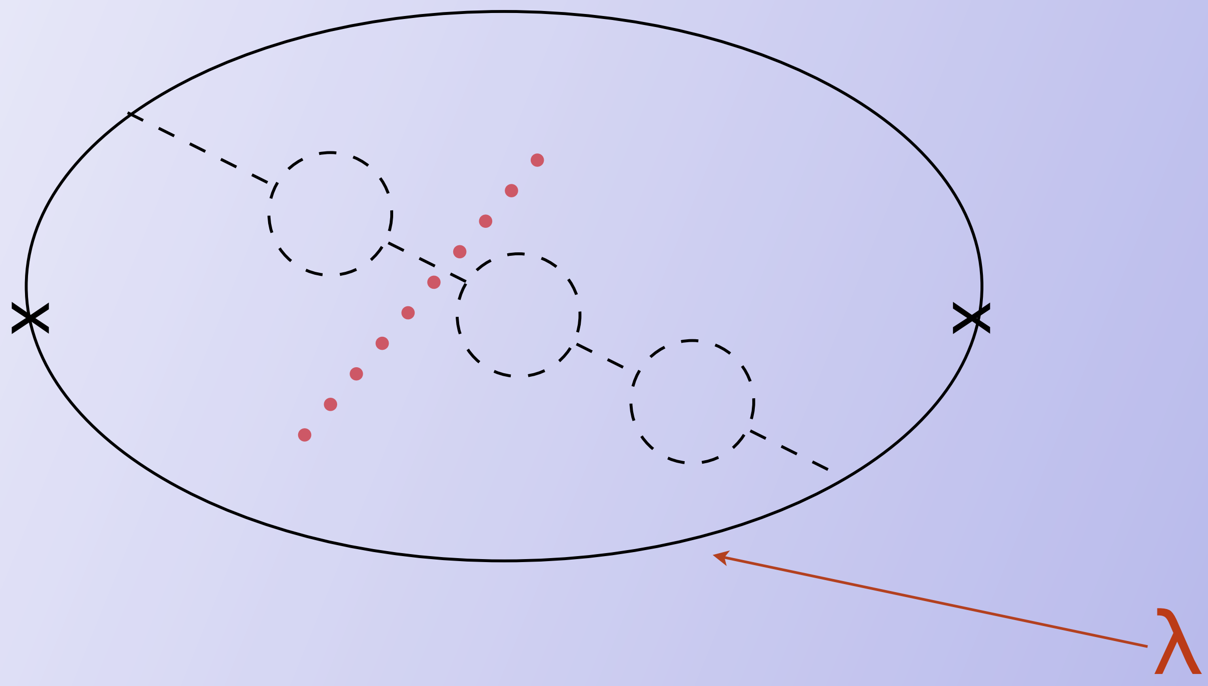
Borel summable

Renormalons and Supersymmetry (SYM)*

*Renormalons and instantons are NOT the only source of singularities in the Borel plane!

$O_{OPE} = G_{\mu\nu}^{a}\, G^{\mu\nu a} \rightarrow$ Borel plane singularity at $a_* = 4$

$LBE_0(g^2) = \int_0^\infty da\, \exp(-a 8\pi^2/\beta_0 g^2)\, f(a)$



$$i \int d^4x\, e^{iqx} \left\langle O(x),\, O^\dagger(0) \right\rangle , \quad O = \mathrm{Tr}\, \bar{\lambda}_{\dot\alpha} \lambda_\alpha \ \text{ or } \ \mathrm{Tr}\, \lambda^\alpha \lambda_\alpha$$

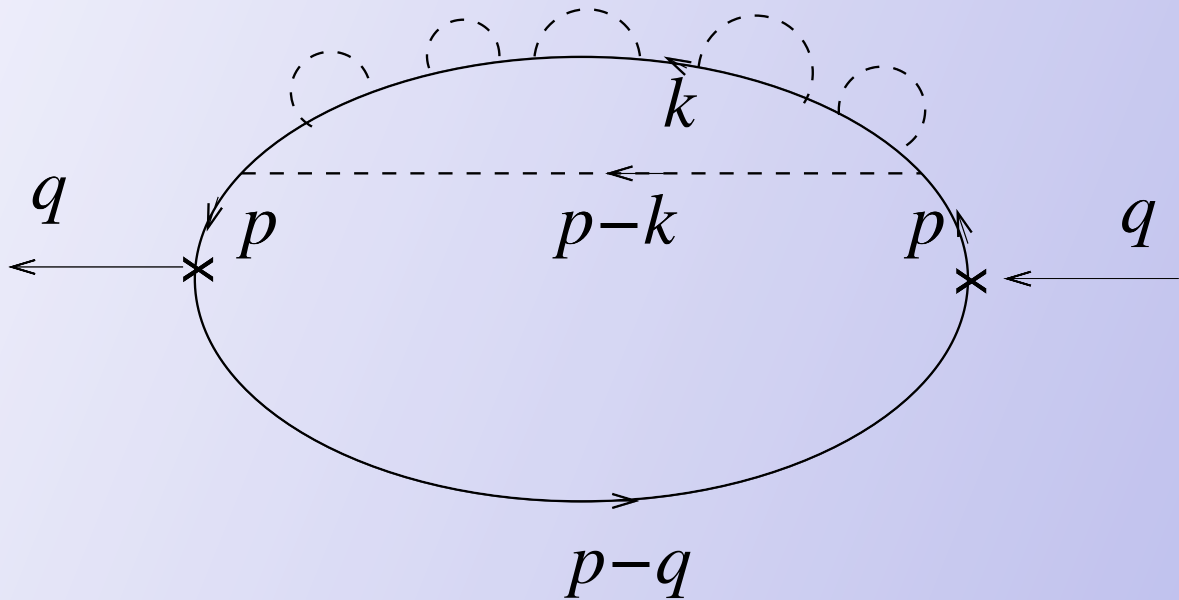
But!..

Theorem: VEV's of all purely gluonic operators vanish in SUSY ☹ ☹ ☹

Non-vanishing-VEV operators contain $\bar{\lambda}^2 \lambda^2$, i.e. start from dim = 6.

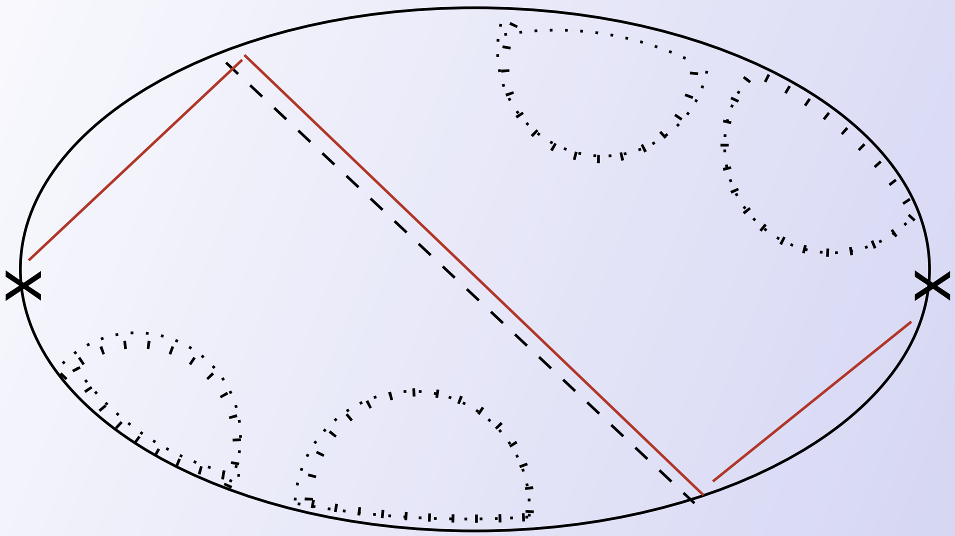
But!..

Using matter bubble trick (as in QCD) we see:

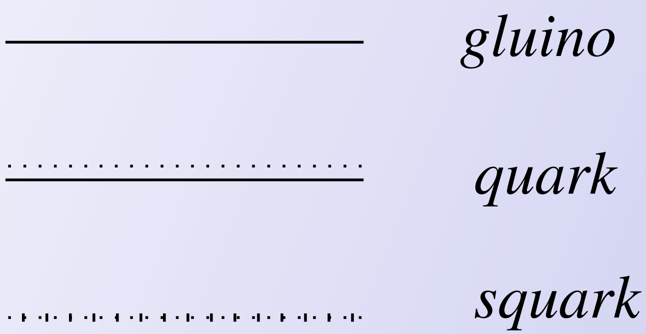


Conjecture:

$$-\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + i\,\bar{\lambda}^a\bar{\sigma}^\mu\,\mathcal{D}_\mu\,,\lambda^a$$



$$O=\lambda^2\bar{\lambda}^2$$



Conclusions

✷ Resurgence in strongly coupled AF FT has subtrleties and is not yet technically achieved;

✷ Conceptual basis is Wilson OPE adapted to QCD (SVZ)

✷ We are at the level of mathematically constructing hyperasymptotics;

✷ IN SYM there are additional technical problems not addressed in the past.