ADVANCED TOPICS IN QUANTUM FIELD THEORY: ERRATA *

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Abstract

I insert a few omitted expressions and correct detected misprints in my book [1].

Introduction

In the process of the editorial preparation of the above book a large number of typos was introduced by typesetters. I managed to fish out most of them in proofreading, but not all. In addition there are some errors for which I am to blame. Below is the list of noted misprints/errors.

Page 38

• The last line in footnote 17 should read:

...four or more derivatives, see [20]. Moreover, if the requirement of scale and Lorentz invariance in four dimensions is supplemented by *unitarity* the class of "abnormal" theories, in which scale invariance does not necessarily entail conformal invariance, is essentially empty. It is likely to be exhausted by theories reducible to free field theories

* Advanced Topics in Quantum Field Theory. A Lectute Course, Cambridge University Press, 2012.

of a special type. (A. Dymarsky, Z. Komargodski, A. Schwimmer, and S. Theisen, *On Scale and Conformal Invariance in Four Dimensions,* arXiv:1309.2921 [hep-th]).

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• Figure 2.22 and the caption below:
The letters χ in Fig. 2.22 must be replaced by ϕ.

• The caption below Fig. 2.22 should read:

Fig. 2.22. The mass parameter renormalization as it follows from renormalization of v^2 due to the one-loop correction to (8.1), see (8.2). This graph yields $v_R^2 = v_0^2 - \frac{3}{4\pi}\,\ln M_{\rm UV}^2$.

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• In Eq. (18.11) the minus sign is misplaced. The correct expression is

$$U_1(\boldsymbol{x}) = \exp\left[-i\pi\,\frac{\boldsymbol{x}\,\boldsymbol{\tau}}{(\boldsymbol{x}^2+\rho^2)^{1/2}}\right], \tag{18.11}$$

• In Eq. (18.12) one should replace f^{abc} by ε^{abc}. The correct expression is

$$K_\mu = 2\varepsilon^{\mu\nu\alpha\beta}\left(A_\nu^a\partial_\alpha A_\beta^a + \frac{g}{3}\,\varepsilon^{abc}A_\nu^a\,A_\alpha^b\,A_\beta^c\right), \tag{18.12}$$

• At the end of the second line after Eq. (18.13) one should add:

(cf. Eqs (16.21) and (16.32))

Page 180: Solution of Exercise 18.1

Let us rewrite Eq. (18.11) as

$$U(\boldsymbol{x}) = \exp\left[-i\pi\, \boldsymbol{\tau}\, \boldsymbol{n}\, \frac{|\boldsymbol{x}|}{(\boldsymbol{x}^2+\rho^2)^{1/2}}\right],$$

where the unit vector $\boldsymbol{n}$ is defined as

$$\boldsymbol{n} = \frac{\boldsymbol{x}}{|\boldsymbol{x}|}\,.$$

Using (18.12) and (18.13) we can write

$$\mathcal{K} = \int d^3x\; \frac{1}{8\pi^2}\,\varepsilon^{ijk}\,\mathrm{Tr}\left(A_i\partial_j A_k - \frac{2i}{3}\,A_iA_jA_k\right), \quad A_i \equiv g\,\frac{\tau^a}{2}A_i^a\,.$$

If we use (18.5) and the identity for the unitary matrices $\partial_j U = -U\left(\partial_j U^\dagger\right)U$ we arrive at

$$\mathcal{K} = \int d^3x\; \frac{1}{24\pi^2}\,\varepsilon^{ijk}\,\mathrm{Tr}\left[\left(U\partial_i U^\dagger\right)\left(U\partial_j U^\dagger\right)\left(U\partial_k U^\dagger\right)\right],$$

where in what follows we will abbreviate $\left(U\partial_i U^\dagger\right) \longrightarrow U\partial_i U^\dagger$.

In order to complete the exercise it is convenient to deal with more general matrices

$$U(\boldsymbol{x}) = \exp\left[-i\,\boldsymbol{\tau}\,\boldsymbol{n}\,F(|\boldsymbol{x}|)\right] \equiv \cos F - i\,\boldsymbol{\tau}\,\boldsymbol{n}\sin F\,,$$

where at the very end we will specify

$$F = \pi\,\frac{|\boldsymbol{x}|}{(\boldsymbol{x}^2+\rho^2)^{1/2}}\,.$$

A few useful relations and notation follow:

$$r \equiv |\boldsymbol{x}|\,, \qquad n_k \equiv x_k/r\,, \qquad \gamma_{kl} = \delta_{kl} - n_k n_l\,,$$

$$U^\dagger = \cos F + i\,\boldsymbol{\tau}\,\boldsymbol{n}\sin F\,,$$

$$\partial_k F = F'\,n_k\,, \qquad F' \equiv \frac{\partial F}{\partial r}\,.$$

Moreover,

$$\partial_k n_l = \frac{1}{r}\,\gamma_{kl}$$

and

$$U\partial_k U^\dagger = i\,\boldsymbol{\tau n}\,F'\,n_k + \sin F\cos F\,\frac{i}{r}\,\gamma_{k\ell}\,\tau_\ell + n_p\,\frac{i}{r}\,\varepsilon^{p\ell m}\,\tau_m\,(\sin F)^2\gamma_{kl}\,.$$

Now we have to take the product $U\partial U^\dagger\,U\partial U^\dagger\,U\partial U^\dagger$ and then the trace. After all differentiations are done we are free to choose the reference frame at any given point in the most convenient way. We will choose

$$\boldsymbol{n} = (0,0,1)\,.$$

Then $\gamma_{k\ell} = 0$ if either k or ℓ =3, and

$$\gamma_{\tilde{k}\tilde{\ell}} = \delta_{\tilde{k}\tilde{\ell}}\,, \qquad \tilde{k}\,,\tilde{\ell} = 1,2\,.$$

Correspondingly,

$$-iA_k = U\partial_k U^\dagger = i\tau_3 F'\delta_{k3} + \frac{i}{r}(\sin F)(\cos F)\tau_{\tilde{k}} + \frac{i}{r}\varepsilon^{\tilde{k}\tilde{m}}\tau_{\tilde{m}}\,(\sin F)^2\,.$$

This implies in turn that in the product $\mathrm{Tr}\,\varepsilon^{ijk}A_iA_jA_k$ the first term can (and must) enter only once being multiplied either by the second term times the second term or by the third term times the third term. As a result, we obtain

$$\int d^3x\,\varepsilon^{ijk}\,\mathrm{Tr}\,\left[U\partial_i U^\dagger\,U\partial_j U^\dagger\,U\partial_k U^\dagger\right] = \int d^3x\,6\,\frac{F'(1-\cos 2F)}{r^2}\,.$$

Now, we can demonstrate that $\mathcal{K}$ reduces to the surface term, namely after performing the angular integration

$$\mathcal{K} = \int_0^\infty dr\,\frac{1}{\pi}\,F'\,(1-\cos 2F) = \frac{1}{\pi}\,\left(F - \frac{1}{2}\sin 2F\right)\Big|_0^\infty = 1\,,$$

quod erat demonstrandum.

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• The following footnote is missing concerning Eq. (21.124):

One may want to check that the first line in Eq. (21.124) vanishes for pure gauge (i.e. for $F = -2/R$). For this check to be carried out one should remember that the integrand contains a full derivative term not presented in (21.124), namely, $(R\,F^2)'$. Incorporating it we add in the first line (inside the brackets) an additional term,

$$\frac{F^2}{R^2} + \frac{2FF'}{R}.$$

Then it becomes immediately clear that the integrand vanishes upon substitution $F \to -2/R$.

Page 247

• The year of publication in Ref. [55], "(1970)",
should be replaced by

• (1969).

Page 263

• The last line in Section 28.2
"Polyakov and Belavin [4]."
should be replaced by

• Polyakov (Physics Letters, **B59**, 79 (1975)).

Page 264

• The following sentence is omitted after Eq. (28.31):
A traditional calculation of the two-loop β function in the CP$(N-1)$ model can be found in D. Friedan, Phys. Rev. Lett. **45**, 1057 (1980); L. Alvarez-Gaumé, D. Z. Freedman, and S. Mukhi, Annals Phys. **134**, 85 (1981).

Page 272

• The last sentence in the paragraph following Eq. (30.21)
"The massless boson is still present ..."
should be replaced by

• The correlation function in (30.19) exhibits a power-like behavior typical of long-range interactions. (To be more exact, (30.19) implying the absence of the mass gap is valid at large F. As one decreases F nonperturbative effects due to vortices present in the Euclidean version of this theory become important, and a mass gap is generated, leading to an exponential fall-off of the two-point function (30.19), as discussed in V. Berezinskii, Sov. Phys. –JETP, **32**, 493 (1971); J. M. Kosterlitz and D. J. Thouless, Journal of Physics **C6**, 1181 1973).)

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• The fifth line
"...more systematic basis."
should be replaced by

• ...more systematic basis. (Only theories with local anomalies will be considered in this Chapter. For a discussion of global anomalies see Section 21.15.)

Page 319

• There are three typos in Eq. (34.10). The correct equation should read

$$\partial^\mu j_\mu^{A,\,R} = \bar\psi_f(x+\varepsilon)\Big[-ig\not{A}(x+\varepsilon)\,\gamma^5 - \gamma^5\,ig\not{A}(x-\varepsilon) + ig\,\gamma^\mu\gamma^5\varepsilon^\beta\,G_{\mu\beta}(x)\Big]\,\psi^f(x-\varepsilon)\,. \tag{34.10}$$

- In the first and second lines in Eq. (34.11) one should replace

$$G_{\rho\mu}(0) \longrightarrow G_{\rho\mu}(x)\,, \qquad \tilde G_{\alpha\phi}(0) \longrightarrow \tilde G_{\alpha\phi}(x)\,.$$

- In Eq. (34.13) one should replace

$$\tilde G_{\alpha\phi}(0) \longrightarrow \tilde G_{\alpha\phi}(x)\,.$$

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Exercise 34.2 was erroneously omitted.

34.2a Prove that if $F_{\mu\nu}$ is x-independent one can always choose a gauge in which

$$A_\mu(x) = \frac{1}{2}x^\rho F_{\rho\mu}\,.$$

Hint: start from the Taylor expansion for A_μ in arbitrary gauge.

34.2b The Green function $S(x,y)$ is defined as $-i\langle T\{\psi(x)\bar\psi(y)\}\rangle$ and satisfies the equation

$$i\not{\mathcal{D}}\,S(x,y) = \delta^{(4)}(x-y)\,. \tag{1}$$

From this equation find the first term in (34.13). then, using

$$A_\mu(x) = \frac{1}{2}x^\rho F_{\rho\mu}\,.$$

as a perturbation in (1), find the second term in (34.13).

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The second line in Eq. (45.22) was erroneously omitted. The correct equation is

$$
\begin{aligned}
&(\sigma^{\mu})_{\alpha\dot\beta}\,(\sigma^{\nu})_{\gamma}^{\ \dot\beta} + (\sigma^{\nu})_{\alpha\dot\beta}\,(\sigma^{\mu})_{\gamma}^{\ \dot\beta} = -2\,g^{\mu\nu}\,\varepsilon_{\alpha\gamma}\,,\\
&(\sigma^{\mu})_{\gamma\dot\beta}\,(\sigma_{\mu})_{\alpha\dot\alpha} = 2\varepsilon_{\gamma\alpha}\,\varepsilon_{\dot\beta\dot\alpha}\,.
\end{aligned}
\tag{45.22}
$$

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The factor $\frac{1}{4}$ in Eq. (E45.1) must be replaced by $\frac{1}{2}$. The correct equation is

$$
\tilde\Psi = \exp\left(-\frac{1}{2}\Sigma^{\mu\nu}\omega_{\mu\nu}\right)\Psi\,, \qquad \Sigma^{\mu\nu} = \begin{pmatrix}\sigma^{\mu\nu} & 0\\ 0 & \bar\sigma^{\mu\nu}\end{pmatrix}.
\tag{E45.1}
$$

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Equation (46.1) is incomplete. The correct equation is

$$
\mathcal{L} = \partial_\mu\varphi_1\partial^\mu\varphi_1 + \partial_\mu\varphi_2\partial^\mu\varphi_2 - m^2\left(\varphi_1^2+\varphi_2^2\right).
\tag{46.1}
$$

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Parentheses are omitted in Eq. (47.25). The correct equation is

$$2\left(j-\frac{1}{2}\right)+1 \quad \text{and} \quad 2\left(j+\frac{1}{2}\right)+1\,. \tag{47.25}$$

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There are three typos in Eq. (E48.2). The correct equation is

$$\begin{aligned}
&\delta C = i(\epsilon\chi - \bar{\epsilon}\bar{\chi}),\\
&\delta\chi_\alpha = \sqrt{2}\,M\epsilon_\alpha + 2i\,v_{\alpha\dot{\alpha}}\,\bar{\epsilon}^{\dot{\alpha}} - (\partial_{\alpha\dot{\alpha}}C)\,\bar{\epsilon}^{\,\dot{\alpha}},\\
&\delta M = 2\sqrt{2}\,\bar{\epsilon}_{\dot{\alpha}}\,\bar{\lambda}^{\dot{\alpha}} - i\sqrt{2}\,\bar{\epsilon}_{\,\dot{\alpha}}\partial^{\alpha\dot{\alpha}}\chi_\alpha\,,\\
&\delta v_{\alpha\dot{\alpha}} = \left[-\frac{1}{2}\epsilon^\beta(\partial_{\beta\dot{\alpha}}\chi_\alpha) + \frac{1}{2}\epsilon_\alpha\partial_{\beta\dot{\alpha}}\chi^\beta - 2i\epsilon_\alpha\bar{\lambda}_{\dot{\alpha}}\right] + \text{H.c.}\\
&\delta\lambda_\alpha = i\,\epsilon_\alpha D + \frac{1}{2}\,\epsilon_\beta\,\partial^{\dot{\alpha}\beta}v_{\alpha\,\dot{\alpha}} - \frac{1}{2}\epsilon^{\,\beta}\,\partial_{\alpha\dot{\beta}}\,v^{\dot{\beta}\beta}\,,\\
&\delta D = \epsilon^\alpha\partial_{\alpha\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}} + \text{H.c.}
\end{aligned} \tag{E48.2}$$

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A term is missing in the third line of Eq. (49.69). The correct equation is

$$\mathcal{L} = \frac{1}{e^2}\left\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar\lambda_{\dot\alpha} i\partial^{\dot\alpha\alpha}\lambda_\alpha\right\}$$
$$+ \left[\mathcal{D}^\mu \bar q\,\mathcal{D}_\mu q + \bar\psi_{\dot\alpha} i\mathcal{D}^{\dot\alpha\alpha}\psi_\alpha\right] + \left[\mathcal{D}^\mu \bar{\tilde q}\,\mathcal{D}_\mu \tilde q + \bar{\tilde\psi}_{\dot\alpha} i\mathcal{D}^{\dot\alpha\alpha}\tilde\psi_\alpha\right]$$
$$+ \left[i\sqrt2\,(\lambda\,\psi)\,\bar q + \text{H.c.}\right] + \left[-i\sqrt2\left(\lambda\,\tilde\psi\right)\bar{\tilde q} + \text{H.c.}\right] + \left(m\psi\tilde\psi + \text{H.c.}\right)$$
$$- V(q\,,\tilde q)\,. \tag{49.69}$$

Page 453

Exercise 52.1. Solution was omitted in typesetting.

- In the second line of this exercise replace (49.72) by (49.74).
- The solution is as follows:

We start from the parametrization

$$q = \sqrt{\xi}\, e^{i\alpha}\cosh\rho\,, \qquad \tilde q = \sqrt{\xi}\, e^{i\alpha}\sinh\rho\,.$$

Then

$$q\tilde q = \frac{\xi}{2}\, e^{2i\alpha}\sinh 2\rho \equiv \frac{\xi}{2}\,\varphi\,, \quad \varphi = e^{2i\alpha}\sinh 2\rho\,, \quad \bar\varphi\varphi = \sinh^2 2\rho\,.$$

The relevant mass term in the Lagrangian is

$$\Delta\mathcal{L}_m = \left[i\sqrt2\,(\lambda\psi)\,\bar q + \text{H.c.}\right] - \left[i\sqrt2\left(\lambda\tilde\psi\right)\bar{\tilde q} + \text{H.c.}\right]. \tag{2}$$

In Eq. (2) we must rescale the field λ,

$$\lambda \to e\,\lambda\,,$$

in order to make its kinetic term canonically normalized. Then, substituting the above expressions for q and $\tilde{q}$ we get

$$\Delta\mathcal{L}_m \longrightarrow e\, i\, \sqrt{2}\, e^{-i\alpha} \left[(\lambda\psi)\, \sqrt{\xi} \cosh\rho - \left(\lambda\tilde{\psi}\right) \sqrt{\xi} \sinh\rho\right] + \text{H.c.} \quad (3)$$

The phase factor $i\, e^{-i\alpha}$ can be absorbed in ψ, $\tilde{\psi}$. We will omit it hereafter.

Let us introduce

$$\eta = \frac{\psi \cosh\rho - \tilde{\psi}\sinh\rho}{\sqrt{\cosh 2\rho}}\,, \qquad \tilde{\eta} = \frac{\psi\sinh\rho + \tilde{\psi}\cosh\rho}{\sqrt{\cosh 2\rho}}\,. \quad (4)$$

Then the kinetic terms of λ, η, and $\tilde{\eta}$ are canonic, i.e.

$$\bar{\lambda}\mathcal{D}\lambda + \bar{\eta}\mathcal{D}\eta + \bar{\tilde{\eta}}\mathcal{D}\tilde{\eta}\,,$$

while the mass term becomes

$$\Delta\mathcal{L}_m = e\, \sqrt{2\xi}\sqrt{\cosh 2\rho}\, \left(\lambda\eta + \bar{\lambda}\bar{\eta}\right). \quad (5)$$

Note that the $\tilde{\eta}$ field stays *massless*. Inspecting Eq. (5) we observe that the diagonal combinations are

$$\lambda_{\pm} = \frac{\lambda \pm \eta}{\sqrt{2}}\,, \qquad \lambda\eta \equiv \frac{1}{4}\left(\lambda_+^2 - \lambda_-^2\right) \quad (6)$$

while the mass for these diagonal combination is

$$e\sqrt{2\xi}\sqrt{\cosh 2\rho} = e\sqrt{2\xi}\,(1 + \bar{\varphi}\varphi)^{1/4}\,. \quad (7)$$

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- The third line after Eq. (53.9) should read:

One linear combination of the photino λ and $\tilde{\psi}$ is the Goldstino; it is massless. Other linear combinations, and the scalar and spinor fields from $\tilde{Q}$ are massive.

- Exercise 53.1. Solution was omitted in typesetting.
- In the second line of this exercise replace (49.74) by (49.68).
- The solution is as follows:

Let us start from the fermion mass matrix. We consider the case $\xi > m^2/e^2$; hence the vacuum fields are $\tilde{q} = 0$ and $q = \left(\xi - \frac{m^2}{e^2}\right)^{1/2}$, where the parameter m is assumed to be real and positive.

The fermion mass term can be extracted[a] from Eq. (49.69). To ensure that the kinetic term of λ is normalized canonically we must replace $\lambda \to e\lambda$. In fact, it is convenient to absorb the phase factor i into λ too,[b] so that the replacement is as follows:

$$\lambda \to -i\, e\, \lambda\,. \tag{8}$$

With this substitution the fermion mass term in the Lagrangian takes the form

$$\Delta\mathcal{L}_m = i\sqrt{2}e\,(\lambda\psi)\sqrt{\xi - \frac{m^2}{e^2}} + m(\tilde{\psi}\psi)\,. \tag{9}$$

This mass term can be represented as the following matrix:

$$\left(\lambda,\, \psi,\, \tilde{\psi}\right)\begin{pmatrix} 0 & \frac{e}{\sqrt{2}}\sqrt{\xi - \frac{m^2}{e^2}} & 0 \\ \frac{e}{\sqrt{2}}\sqrt{\xi - \frac{m^2}{e^2}} & 0 & \frac{m}{2} \\ 0 & \frac{m}{2} & 0 \end{pmatrix}\begin{pmatrix} \lambda \\ \psi \\ \tilde{\psi} \end{pmatrix}. \tag{10}$$

This mass matrix can be easily diagonalized. It has one zero eigenvalue and two nonvanishing eigenvalues. The massless Goldstino corresponding to the vanishing eigenvalue is

$$\left(\frac{2e^2\xi}{m^2} - 1\right)^{-1/2}\left[\lambda - \left(\frac{2e^2\xi}{m^2} - 2\right)^{1/2}\tilde{\psi}\right], \tag{11}$$

where I included the normalization factor in front of the square brac-

[a] See also Erratum to page 440.

[b] If this purely imaginary factor were not absorbed, the derivation of the fermion mass spectrum would be somewhat more complicated; the mass matrix to be considered would have to include not only λ, ψ and $\tilde{\psi}$ but also complex conjugated of these fields. It would be 6 by 6, rather than 3 by 3. Needless to say, the final answer will be the same.

kets. Two diagonal combinations with nonvanishing mass terms are

$$\frac{1}{\sqrt{2}}\,\frac{1}{\sqrt{\xi-\frac{m^2}{2e^2}}}\left(\sqrt{\xi-\frac{m^2}{e^2}}\,\lambda\pm\sqrt{\xi-\frac{m^2}{2e^2}}\,\psi+\frac{m}{\sqrt{2}e}\tilde{\psi}\right) \tag{12}$$

where the corresponding mass terms are

$$\pm\frac{e}{\sqrt{2}}\sqrt{\xi-\frac{m^2}{2e^2}}\,. \tag{13}$$

• Now let us analyze the boson masses. Since in the vacuum $\tilde{q}=0$, fluctuations of this field coincide with the field itself. For the field q we write

$$q=\sqrt{\xi-\frac{m^2}{e^2}}+\delta q\,, \tag{14}$$

(δq is real!). We start from the scalar potential in (49.68) and expand it (up to quadratic terms) in δq and $\tilde{q}$,

$$V(\delta q,\,\tilde{q})=\mathcal{E}_{\rm vac}+2e^2\left(\sqrt{\xi-\frac{m^2}{e^2}}\right)(\delta q)^2+2m^2|\tilde{q}|^2+...,\tag{15}$$

where the ellipses denote cubic and higher order terms. From (15) we see that the mass of the δq quantum is (this is a real field)

$$m(\delta q)=\sqrt{2}\,e\sqrt{\xi-\frac{m^2}{2e^2}}\,, \tag{16}$$

while that of the complex field $\tilde{q}$ is

$$m(\tilde{q})=\sqrt{2}\,m\,. \tag{17}$$

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• The authors of the third paper in [43] are M. Grisaru, M. Roček, and W. Siegel.

Page 591

• At the end of the Section 72.7 a reference is omitted, P. A. Bolokhov, M. Shifman and A. Yung, 2*D*-4*D* *Correspondence: Towers of Kinks versus Towers of Monopoles in* $\mathcal{N} = 2$ *Theories*, Phys. Rev. D **85**, 085028 (2012). This paper discusses CMS in CP$(N-1)$ models with $N > 2$.

References

1. M. Shifman, *Advanced Topics in Quantum Field Theory. A Lectute Course*, (Cambridge University Press, 2012).