Lecture 14. Binomial Trees continued

Introduction to Binomial Trees
Problem 9.22: A European call option and put option on a stock both have a strike price of $20 and an expiration date in 3 months. Both sell for $3. The risk-free interest rate is 10% per annum, the current stock price is $19, and a $1 dividend is expected in 1 month. Identify the arbitrage opportunity open to a trader.

Solution:

Here we can use the put-call parity to decide if either the put or the call is overpriced. We have

\[ c + Ke^{-rT} + De^{-rT_0} = p + S_0 \]

where \( T \) is the expiration date of the options and \( T_0 \) is the maturity date of the dividend.

Plugging in for the put price yields

\[ p = c + Ke^{-rT} + De^{-rT_0} - S_0 \]

\[ = 3 + 20e^{-0.10 \times \frac{3}{12}} + 1e^{-0.10 \times \frac{1}{12}} - 19 \]

\[ = 4.50 \]

Therefore, the put is underpriced relative to what it should be. We should therefore buy the put and buy the stock simultaneously. On the other hand we should write the call (since relatively overpriced and exists comparative advantage).
Solutions

Portfolio consists of

1. Long put costs $3
2. Long stock costs $19
3. Short call pays $3

Therefore, we pay $3 + $19 - $3 = $19 for the deal. In one month we earn $1 from the stock. In 3 months we have two options

- If $S_T > 20$ then our put is worthless. The call is exercised for $20, and we sell our stock. We make $20 + 1 = 21$.
- If $S_T < 20$ then we exercise the put and sell our stock for $20. The call is worthless. We earn $20 + 1$.

Present value of the arbitrage is

$$-3 - 19 + 3 + 20e^{-0.10 \times \frac{3}{12}} + e^{-0.10 \times \frac{1}{12}} = 1.50$$
Problem 9.23: Suppose that \( c_1, c_2, \) and \( c_3 \) are the prices of European call options with strike prices \( K_1, K_2, \) and \( K_3, \) respectively, where \( K_3 > K_2 > K_1 \) and \( K_3 - K_2 = K_2 - K_1. \) All options have the same maturity. Show

\[
c_2 \leq 0.5 (c_1 + c_3)
\]

Solution: Use the hint:

Consider one portfolio that is long one option with strike price \( K_1, \) long one option with strike price \( K_3 \) and short two options with strike price \( K_2. \)

This is the butterfly spread! Consider the following cases:

- If \( S_T < K_1 \) then the portfolio is worth

\[
0 - 2 \times 0 + 0 = 0
\]
Solutions

- If $K_1 \leq S_T \leq K_2$ then the portfolio is worth
  
  $$(S_T - K_1) - 2 \times 0 + 0 = S_T - K_1$$

- If $K_2 \leq S_T \leq K_3$ then the portfolio is worth
  
  $$(S_T - K_1) - 2 \times (S_T - K_2) + 0 = 2K_2 - K_1 - S_T = K_3 - S_T$$

- If $K_3 \leq S_T$ then the portfolio is worth
  
  $$(S_T - K_1) - 2 \times (S_T - K_2) + (S_T - K_3) = 2K_2 - (K_1 + K_3) = 0$$

Therefore, since the butterfly spread (ignoring the transaction costs) is always positive or zero, we have that the portfolio should have non-negative worth. Hence,

$$c_1 - 2c_2 + c_3 \geq 0$$

or

$$c_2 \leq \frac{1}{2} (c_1 + c_3)$$
Problem 9.24: What is the result corresponding to that in Problem 9.23 for European put options?

Solution: Again consider one portfolio that is long one option with strike price $K_1$, long one option with strike price $K_3$ and short two options with strike price $K_2$.

- If $S_T < K_1$ then the portfolio is worth
  \[
  (K_1 - S_T) - 2 \times (K_2 - S_T) + (K_3 - S_T) = K_1 + K_3 - 2K_2 = 0
  \]

- If $K_1 \leq S_T \leq K_2$ then the portfolio is worth
  \[
  0 - 2 \times (K_2 - S_T) + (K_3 - S_T) = S_T - 2K_2 + K_3 = S_T - 2K_2 + K_3 = S_T - K_1 \geq 0
  \]

- If $K_2 \leq S_T \leq K_3$ then the portfolio is worth
  \[
  0 - 2 \times 0 + (K_3 - S_T) = K_3 - S_T \geq 0
  \]

- If $K_3 \leq S_T$ then the portfolio is worth
  \[
  0 - 2 \times 0 + 0 = 0
  \]
Solution

Again the portfolio is always positive, hence $p_1 - 2p_2 + p_3 \geq 0$. Therefore,

$$p_2 \leq \frac{1}{2} (p_1 + p_3)$$
Binomial trees are useful tools for pricing options. Construct charts of possible movements of a stock and price according to the movements.

We assume that the stock price is a random walk, i.e. at each time step there is a certain probability of the stock moving in one direction or another (up or down).
Consider now a stock with price $S_0$ and an option with current price $f$.

- Suppose that the option lasts for time $T$ and during the life of the option, the price can go either up from $S_0$ to $S_0u$ (where $u > 1$) or down from $S_0$ to $S_0d$ (where $d < 1$).

- The percentage increase in the stock price in up movement is $u - 1$. The percentage decrease in the stock price in down movement is $1 - d$.
- Let the option payoff for up movement be $f_u$ and the option payoff for down movement be $f_d$.
Generalization

- Build a portfolio of a long position in $\Delta$ shares and a short position in one option.
- On up movement the value of the portfolio is at the end of the option
  \[ S_0u\Delta - f_u \]
- On down movement the value of the portfolio is at the end of the option
  \[ S_0d\Delta - f_d \]
- Equality if
  \[ S_0u\Delta - f_u = S_0d\Delta - f_d \]  \hspace{1cm} (1)
  or
  \[ \Delta = \frac{f_u - f_d}{S_0u - S_0d} \]
  Thus $\Delta$ is the ratio of the change in the options prices to the change in the stock prices.
- Portfolio is now risk-less, and so earns risk-free rate $r$. Present value of the portfolio (due to (1)) is
  \[ (S_0u\Delta - f_u) e^{-rT} \]
Generalization

- Cost of setting up the portfolio is
  \[ S_0 \Delta - f \]

- Since cost should equal present value (else arbitrage opportunity)
  \[ S_0 \Delta - f = (S_0 u \Delta - f_u) e^{-rT} \]
  or
  \[ f = S_0 \Delta \left( 1 - ue^{-rT} \right) + f_u e^{-rT} \]

- Or
  \[ f = e^{-rT} \left[ pf_u + (1 - p) f_d \right] \]
  where
  \[ p = \left( \frac{e^{rT} - d}{u - d} \right) \]
  Or
  \[ p = \left( \frac{a - d}{u - d} \right) \quad a = e^{rT} \]

Remark: Pricing independent of the probability of the stock moving up or down! Calculating the price in terms of the underlying stock (incorporated in the value of the stock price already).
Risk-Neutral Valuation

• Natural to interpret \( p = \frac{e^{rT} - d}{u - d} \) as the probability of the upward movement. Then \( 1 - p \) is the probability of downward movement.

• Then

\[
p f_u + (1 - p) f_d
\]

is the expectation (expected payoff = probability \( \times \) payout).

• In a similar vein, if we consider the same probability of up and down movement, then the expectation of stock value should be

\[
p S_0 u + (1 - p) S_0 d = \frac{e^{rT} - d}{u - d} S_0 u + \left( 1 - \frac{e^{rT} - d}{u - d} \right) S_0 d
\]

\[
= \frac{e^{rT} - d}{u - d} S_0 u + \frac{u - e^{rT}}{u - d} S_0 d
\]

\[
= S_0 \left[ \frac{ue^{rT} - ud + ud - d e^{rT}}{u - d} \right]
\]

\[
= S_0 e^{rT}
\]
Risk-Neutral Valuation

• Therefore, a stock should grow at the risk-free rate on average. This is an example of a risk-neutral valuation.

• A risk-neutral world is a market in which all individuals are indifferent to risk. Investors require no compensation for risk. All stocks gain at the risk-free rate.

• Risk-neutral valuation says we can assume the world is risk-neutral when pricing options.
Real World vs. Risk-Neutral World

- The probability \( p \) of up movement is from a risk-neutral world. Then

\[
E(S_T) = S_0 e^{rT}
\]

- Suppose on the contrary the expected return is 16\% and \( p^* \) is the probability of up movement in the real world. Then from our first example with \( S_0 = 20 \), \( S_0 u = 22 \), and \( S_0 d = 18 \), then

\[
20e^{0.16 \times \frac{3}{12}} = [22p^* + 18(1 - p^*)]
\]

then

\[
4p^* = 20e^{0.16 \times \frac{3}{12}} - 18
\]

or

\[
p^* = 0.7041
\]

- We interpret this as the expected payoff from the option in the real world is

\[
p^* \times f_u + (1 - p^*) \times f_d = p^* = 0.7041
\]

since \( f_u = 1 \) and \( f_d = 0 \).

- **However**, it is riskier to own the option than the stock, so the discount rate should be higher than 16\% to compensate for this. How to quantify??

- Easier to use the risk-neutral world assumptions to calculate the prices.
We can extend the analysis of one-step trees to two-step trees. Consider a stock with a price at $20. In each time step of 3 months, the price can go up 10% or down 10%. Suppose the risk-free rate is 12% per annum. Assume the option price is $21.

We would like to compute the price of a call option given the above data.

Method? Work **backwards**!
Price option for generalization. Set time scale to be $\Delta t$.

- Price is

$$f = e^{-r\Delta t} [pf_u + (1 - p)f_d]$$

where

$$p = \frac{e^{rT} - d}{u - d}$$
Generalization

• Again price is

\[ f = e^{-r\Delta t} [pf_u + (1 - p)f_d] \]

• For the node at \( S_0u \) price is

\[ f_u = e^{-r\Delta t} [pf_{uu} + (1 - p)f_{ud}] \]

• For the node at \( S_0d \) price is

\[ f_d = e^{-r\Delta t} [pf_{du} + (1 - p)f_{dd}] \]

• Then substituting we get

\[
\begin{align*}
    f &= e^{-r\Delta t} [pf_u + (1 - p)f_d] \\
    &= e^{-r\Delta t} \left[ p \left( e^{-r\Delta t} [pf_{uu} + (1 - p)f_{ud}] \right) + (1 - p) \left( e^{-r\Delta t} [pf_{du} + (1 - p)f_{dd}] \right) \right] \\
    &= e^{-2r\Delta t} \left[ p^2 f_{uu} + 2p(1 - p)f_{ud} + (1 - p)^2 f_{dd} \right]
\end{align*}
\]

• The probabilities \( p^2 \), \( 2p(1 - p) \), \( (1 - p)^2 \) are the probabilities of achieve final stock prices of \( S_0u^2 \), \( S_0ud \), \( S_0d^2 \), respectively.

• Risk-neutral evaluation.
Binomial Trees to Price American Options

So far we've evaluated prices for European options. We follow the following procedure to price

1. Final nodes are priced as for European options
2. Previous nodes we price via taking the greater of
   (a) Value given by
   \[
   f = e^{-r\Delta t} \left[ pf_u + (1 - p) f_d \right]
   \] (2)
   (b) Value given by early exercise
Define \textbf{Delta}, $\Delta$, to be the ratio of the change in the price of the stock option to the change in the price of the underlying stock.

Number of units of stock that we should hold for each option shorted in order to create a risk-less hedge.

- A risk-less hedge is referred to as \textit{delta hedging}.

- Tells investor how to adjust the amount of stock in portfolio so that the portfolio remains risk-less.

- \textbf{Example}: First example in the one-step binomial tree, $f_u - f_d = 1 - 0 = 1$ and $S_0u - S_0d = S_0(u - d) = 20(1.1 - 0.9) = 20 \times 0.2 = 4$. Then $\Delta = \frac{1}{4} = 0.25$. 
Delta

- **Example 2**: Consider the second example

  Delta hedge on first step is \(\frac{2.0257 - 0}{22 - 18} = 0.5064\)

  Delta for stock price movements over second time step are

  \[
  \frac{3.2 - 0}{24.2 - 19.8} = 0.7273 \uparrow \quad \frac{0 - 0}{19.8 - 16.2} = 0 \downarrow
  \]

  for up and down movements, respectively.
• **Example 3:** Consider the second example

![Diagram showing stock price movements over two time steps]

- Delta hedge on first step is \( \frac{1.4147 - 9.4636}{60 - 40} = -0.4024 \)
- Delta for stock price movements over second time step are

\[
\frac{0 - 4}{72 - 48} = -0.1667 \uparrow \quad \frac{4 - 20}{48 - 32} = -1.000 \downarrow
\]

for up and down movements, respectively.
Delta

- Therefore, Delta changes over time.

- In order to maintain a risk-less hedge using an option and underlying stock, the stock holdings must be adjusted periodically.
Matching Volatility with $u$ and $d$

When constructing binomial trees, choose $u$ and $d$ to match the volatility of the stock price.

We present the following method:

- Let $\mu$ be the **expected return** on the stock (in the real world).

- Let $\sigma$ be the **volatility**. The volatility $\sigma$ of a stock price is defined so that $\sigma \sqrt{\Delta t}$ is the **standard deviation** of the return on the stock price in a short period of time of length $\Delta t$. More later.

- Let $\Delta t$ be the **interval of time**.

- Suppose the stock price starts at $S_0$ and stock moves to either $S_0u$ or $S_0d$ at the end of $\Delta t$. 
Matching Volatility with $u$ and $d$

- Probability of up movement is $p^*$. Then the expected return at the first time step is
  
  $$S_0e^{\mu\Delta t}$$
  
  and the expected stock price at this time is
  
  $$p^* S_0u + (1 - p^*) S_0d$$
  
  which should match the return. Then
  
  $$p^* S_0u + (1 - p^*) S_0d = S_0 e^{\mu\Delta t}$$
  
- Therefore,
  
  $$p^* = \frac{e^{\mu\Delta t} - d}{u - d}$$
  
- Now we use the volatility into the picture. Since the volatility $\sigma \sqrt{\Delta t}$ is the standard deviation of the return on the stock price over short periods of time $\Delta t$, then $\sigma^2 \Delta t$ is the variance on the return on the stock price over short periods of time.
Matching Volatility with $u$ and $d$

- But the variance on the return is $E(X^2) - (E(X))^2$ for $X = u, d$ then

$$
\sigma^2 \Delta t = p^* u^2 + (1 - p^*) d^2 - (p^* u + (1 - p^*) d)^2
$$

Using

$$
p^* = \frac{e^{\mu \Delta t} - d}{u - d}
$$

then

$$
\sigma^2 \Delta t = p^* u^2 + (1 - p^*) d^2 - (p^* u + (1 - p^*) d)^2
$$

$$
= (u - d)^{-2} \left[ \left( e^{\mu \Delta t} - d \right) (u - d) u^2 + \left( u - e^{\mu \Delta t} \right) (u - d) d^2 \right]
$$

$$
- (u - d)^{-2} \left[ \left( e^{\mu \Delta t} - d \right) u + \left( u - e^{\mu \Delta t} \right) d \right]^2
$$

$$
= e^{\mu \Delta t} (u + d) - ud - e^{2\mu \Delta t}
$$

For $\Delta t$ small, approximate $e^{\mu \Delta t}$, and we can solve for $u$ and $d$:

$$
u = e^{\sigma \sqrt{\Delta t}}
$$

$$
d = e^{-\sigma \sqrt{\Delta t}}
$$
Matching Volatility with $u$ and $d$ 

When we move from the real world to the risk-neutral world then

- The expected return on the stock changes
- The volatility remains the same (as $\Delta t$ goes to zero)
- Known as Giranov’s Theorem. More later.

Moving from one set of risk preferences to another is referred to as changing the measure.

- The real-world measure is the P-measure - $p^*$.
- The risk-neutral measure is the Q-measure - $p$. 
Matching Volatility with $u$ and $d$

Example: Consider the American put option where the stock price is $50$, the strike price is $52$, the risk-free rate is $5\%$, the life of the option is $2$ years, two time steps. Suppose the volatility is $30\%$.

$$u = e^{0.3 \times 1} = 1.3499 \quad d = \frac{1}{1.3499} = 0.7408 \quad e^{rT} = e^{0.05 \times 1} = 1.0513$$

Then $p = \frac{1.053 - 0.7408}{1.3499 - 0.7408} = 0.5097$.

Value of the put is $7.43$. 

Options, Futures, Derivatives 10/21/07  back to start
Matching Volatility with $u$ and $d$

Compare with

Different from the value with $u = 1.2$ and $d = 0.8$ of 4.1923.
Increasing the number of steps

Too little information to get an accurate price of the option. In practice, the life of option divided into many more time steps (as many as 30 steps for $2^{30} \approx 10^9$ possible pathways).

Still many of the variables are the same including

$$u = e^{\sigma \sqrt{\Delta t}} \quad d = e^{-\sigma \sqrt{\Delta t}}$$

and the risk-free rate $r$ and $\Delta t$ over the life of the binomial tree.
Suppose a stock pays a dividend with yield at a rate $q$.

- Total return from the dividends and capital gains in risk neutral world is $r$.
- Dividends provide a return rate of $q$.
- Capital gains is $r - q$.
- Stock starts at $S_0$ then expected value after one time step of $\Delta t$ must be $S_0 e^{(r-q)\Delta t}$. Thus

$$pS_0 u + (1 - p)S_0 d = S_0 e^{(r-q)\Delta t}$$

hence

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d}$$

- Match volatility by setting $u = e^{\sigma \sqrt{\Delta t}}$ and $d = \frac{1}{u}$.
- We have

$$f = e^{-r\Delta t} \left[ pf_u + (1 - p) f_d \right] \quad p = \frac{a - d}{u - d}$$

where

$$a = e^{(r-q)\Delta t} \text{ is the growth factor}$$
Stock index yield example

The risk-neutral pricing model for binomial trees applied to Stock Index Options is similar. If the dividend yield rate is \( q \) then:

\[
f = e^{-r\Delta t} \left[ pf_u + (1 - p)f_d \right]
\]

where

\[
p = \frac{a - d}{u - d}
\]

and \( a = e^{(r-q)\Delta t} \).

**Example**: Consider a stock index at 700 and has volatility of 20\% and a dividend yield of 3\%. The risk-free rate is 7\%. Compute the value of a 3 month American Put option with a strike price of 705 using a 3 step binomial tree.

Note that \( u = e^{\sigma \sqrt{\Delta t}} = e^{0.2 \times \sqrt{\frac{1}{12}}} = 1.0594 \) and \( d = \frac{1}{u} = 0.9439 \). The growth factor \( a = e^{(r-q)\Delta t} = e^{(0.07-0.03) \times \frac{1}{12}} = 1.0033 \). We compute the probabilities as

\[
p = \frac{1.0033 - 0.9439}{1.0594 - 0.9439} = 0.5146
\]

Finally, \( e^{-r\Delta t} = e^{-0.07 \times \frac{1}{12}} = 0.9942 \).

Continued on blackboard.
Options on Stock Indices

- Futures prices on stock index use the yield rate \( q \). Use the same pricing model

\[
f = e^{-r\Delta t} \left[ pf_u + (1 - p) f_d \right] \quad p = \frac{a - d}{u - d}
\]

- Growth factor \( a \) satisfies

\[
a = e^{(r-q)\Delta t}
\]

**Example:** Consider a stock index at 810 with volatility 20% and a dividend yield of 2%. Risk free rate is 5%. Price a 6-month European put option with strike price of 800 using a two step tree.

In order to compute \( u \) and \( d \) we use the volatility: \( u = e^{\sigma \sqrt{\Delta t}} = e^{0.2 \sqrt{\frac{3}{12}}} = 1.1052 \) and \( d = \frac{1}{u} = 0.9048 \). Next we have \( p = \frac{a - d}{u - d} = \frac{e^{(0.05-0.02)\times0.02} - 0.9048}{1.1052 - 0.9048} = 0.5126 \).

- Start with the current price 810 at node A.
- After 3 months, either price goes up to \( 810 \times 1.1052 = 895.19 \) at node B or goes down to \( 810 \times 0.9048 = 732.92 \) at node C.
Options on Stock Indices

- At the end of 6 months, either price goes to $895.19 \times 1.1052 = 989.34$ at node D, goes to price $895.19 \times 0.9048 = 810$ at node E, or goes to $732.92 \times 0.9048 = 663.17$ at node E.

We evaluate the options at the end time. Since a put, then $f_{uu} = 0$ at node D, $f_{ud} = 0$ at node E, and $f_{dd} = 800 - 663.17 = 136.83$.

- At node B we compute

  $f_u = e^{-0.5 \times 0.25} \left[ 0.5126 \times 0 + (1 - 0.5126) \times 0 \right] = 0$

- At node C we compute

  $f_d = e^{-0.5 \times 0.25} \left[ 0.5126 \times 0 + (1 - 0.5126) \times 136.83 \right] = 58.8545$

- At node A we compute

  $f_d = e^{-0.5 \times 0.25} \left[ 0.5126 \times 0 + (1 - 0.5126) \times 58.8545 \right] = 25.315$

Price of the option should be $25.32$
Pricing currency options with binomial trees

Recall a foreign currency can be regarded as an asset providing a yield at a foreign risk-free rate of interest $r_f$. Then rate of growth is $a = e^{(r-r_f)\Delta t}$.

**Example:** Australian dollar is currently 0.6100 USD and the exchange rate has volatility of 12%. The Australian risk-free rate is 7% and the US risk-free rate is 5%. Consider a 3 month American Call option with strike price 0.6000 with a three step tree.

Here $\Delta t = \frac{1}{12} = 0.08333$, $u = e^{\sigma \sqrt{\Delta t}} = e^{0.12 \times \sqrt{0.0833}} = 1.0352$ and $d = \frac{1}{u} = 0.9660$.

Therefore, $a = e^{(r-r_f)\Delta t} = e^{(0.05-0.07)\times \frac{1}{12}} = 0.9983$, so $p = \frac{0.9983-0.9660}{1.0352-0.9660} = 0.4673$. We also have $e^{-0.05\times \frac{1}{12}} = 0.9958$. 
Options on Futures

It does not cost anything to enter into a futures contract on either side (unlike stock). Therefore, if $F_0$ is the initial futures price, the expected futures price at the end of one time step of length $\Delta t$ should also be $F_0$. This implies

$$pF_0u + (1 - p)F_0d = F_0$$

or

$$p = \frac{a - d}{u - d} \text{ with growth rate } a = 1.$$ 

Repeat the pricing argument.

Example:

- Consider a futures price at $31$ and volatility of $30\%$. The risk-free rate is $5\%$. Compute the price of a 9-month American put option with strike price of $30$ using a three step tree.

- Compute $u = e^{0.30 \times \sqrt{3/12}} = 1.1618$ and $d = \frac{1}{u} = 0.8607$. Since $a = 1$ then

$$p = \frac{1 - 0.8607}{1.1618 - 0.8607} = 0.4626$$

and

$$e^{-r\Delta t} = e^{-0.05 \times 0.25} = 0.9876$$
Summary

• Riskless portfolios earn the risk-free interest rate. This enables the stock option to be priced in terms of the stock movements.

• Multistep binomial trees can be resolved by working backwards. No assumptions are required on the probabilities of the up and down movements of the stock at each node.

• Delta of a stock option considers the effect of a small change in the underlying stock price on the change in the option price.

• Constructing binomial trees for valuing options on stock indices, currencies, and futures contracts is very similar to valuing options on stocks.
Midterm: General concepts to concentrate on

- Basic mechanics of the Futures and Options Markets
- Basic differences between Futures and Forward Contracts
- **Differences in continuous vs. periodic compounding interest rates**
- Determining zero rates. Determining bond prices.
- **Determining forward rates** \( R_F = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1} \).
- Basic use of duration in bond prices
- **Determining forward/futures prices** for
  - investment assets
  - investment assets with known income
  - stock indices
  - foreign currency contracts
  - assets providing known yield
  - assets with cost of carry
- Using swaps to take advantage of interest rates (comparative advantage). Valuation
- Currency swaps
- Basic properties of options prices. **Put-call parity**.
- Bull, Bear, Butterfly strategies of stock options
- **Binomial Trees**
Wiener Processes and Itô’s Lemma

More sophisticated approach to modeling the behavior of assets underlying derivatives - view motion as a stochastic process.

A stochastic process is a process where future evolution is described by probability distributions.

Two types: discrete-time stochastic process changes values at discrete time steps. A continuous-time stochastic process changes value at any time.

Stochastic process can be continuous variable or discrete variable. A continuous-variable process can take any value within a certain range. (motion of a particle in fluid). A discrete-variable process can take only certain prescribed values. (coin flips)

Markov Process is a stochastic process where only present value of a variable is relevant for predicting the future. Coin flips are Markovian. If we flip the coin 30 times and comes up heads 30 straight times, next flip still 50/50 chance.