Lecture 9. Swaps

Introduction to Swaps
Swaps

• A swap is an agreement between two companies to exchange cash flows in the future.
• An agreement includes the dates when the cash flows are paid and the way in which they are calculated.
• Most common swap is a plain vanilla swap: One company agrees to pay cash flows equal to interest at a predetermined fixed rate on a notional principal for a number of years.
• In return, the other company pays interest at a floating rate on the same notional principal for the same period of time.
• Most common floating rate is the LIBOR. Typically 1-month, 3-month, 6-month, and 12-month rates.

Example: Consider 5-year bond with a rate of interest specified as 6-month LIBOR plus 0.5% per annum.

• Life the the bond is divided into 10 6-month periods.
• Each period has a rate of interest set at 0.5% per annum above the 6-month LIBOR rate at the beginning of the period.
• Interest is paid at the end of the period.
Consider a 3-year swap initiated on March 5, 2004 between Microsoft and Intel.

- Microsoft pays Intel an interest rate of 5% per annum on a notional principal of $100 million. (fixed-rate payer)
- Intel pays Microsoft the 6-month LIBOR rate on the same notional principal. (floating-rate payer)
- Payments are exchanged every 6 months with 5% interest rate compounded semiannually.
- First payment exchange takes place on Sept. 5, 2004, 6-months after initiation. Microsoft pays Intel $2.5 million. Intel would pay interest at the 6-month LIBOR rate prevailing 6-months prior to Sept. 5, 2004 (March 5, 2004).
- If the 6-month LIBOR rate on Mar. 5, 2004 is 4.2%, then Intel pays Microsoft

\[
0.5 \times 0.042 \times 100,000,000 = 2.1\text{million}
\]
In depth Example, swap

- Second payment exchange takes place on Mar. 5, 2005, 12 months after initiation. Microsoft pays Intel $2.5 million. Intel would pay interest at the 6-month LIBOR rate prevailing 6-months prior to Mar. 5, 2005 (Sept. 5, 2004).
- If the 6-month LIBOR rate on Mar. 5, 2004 is 4.8%, then Intel pays Microsoft

$$0.5 \times 0.048 \times \$100,000,000 = \$2.4 \text{ million}$$

In sum over 6 payments satisfying:

<table>
<thead>
<tr>
<th>Date</th>
<th>Six Month LIBOR rate</th>
<th>Floating cash flow received</th>
<th>Fixed cash flow paid</th>
<th>Net cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mar. 5, 2004</td>
<td>4.20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sept. 5, 2004</td>
<td>4.80</td>
<td>+2.10</td>
<td>-2.50</td>
<td>-0.40</td>
</tr>
<tr>
<td>Mar. 5, 2005</td>
<td>5.30</td>
<td>+2.40</td>
<td>-2.50</td>
<td>-0.10</td>
</tr>
<tr>
<td>Sept. 5, 2005</td>
<td>5.50</td>
<td>+2.65</td>
<td>-2.50</td>
<td>+0.15</td>
</tr>
<tr>
<td>Mar. 5, 2006</td>
<td>5.60</td>
<td>+2.75</td>
<td>-2.50</td>
<td>+0.25</td>
</tr>
<tr>
<td>Sept. 5, 2006</td>
<td>5.90</td>
<td>+2.80</td>
<td>-2.50</td>
<td>+0.30</td>
</tr>
<tr>
<td>Mar. 5, 2007</td>
<td>5.95</td>
<td></td>
<td>-2.50</td>
<td>+0.45</td>
</tr>
</tbody>
</table>
Swaps and Liabilities (Loans)

Swaps can change the nature of loans.

In our example Microsoft can use the swap to transform a floating-rate loan into a fixed-rate loan. Suppose Microsoft has borrowed $100 million at LIBOR plus 10 basis points. Then Microsoft

- pays LIBOR plus 0.1% to its outside lender
- earns LIBOR under the terms of the swap
- pays 5% under terms of the swap

In total Microsoft pays fixed interest payments of 5.1%
Swaps and Assets

Swaps can change the nature of assets.

The swap can change an asset earning a fixed rate of income into an asset that earns a floating rate of interest. Suppose Microsoft owns $100 million in bonds that will earn 4.7% interest per annum over 3 years. After the swap it

- earns 4.7% on the bonds
- earns LIBOR under the terms of the swap
- pays 5% under the terms of the swap

In total Microsoft earns LIBOR minus 30 basis points.
Role of Financial Intermediary

- Non-financial companies usually do not get in touch with each other to arrange swaps.
- Rather companies deal through a financial intermediary.
- Plain-vanilla fixed-point swaps are structured so that financial institutions earn about 3 or 4 basis points on a pair of offsetting transactions per year.

In our example, the financial company setting up the Intel-Microsoft swap would expect to make $0.0003 \times 100 \text{ million} = $30,000$ per year.

Half of the proceeds from each company.

Microsoft would borrow at 5.115% instead of at 5.1%.

Intel would borrow at LIBOR plus 21.5 basis points instead of LIBOR plus 20 basis points.
Many large financial institutions act as market makers for swaps, since unlikely that two companies have completely compatible needs.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Bid</th>
<th>Offer</th>
<th>Swap rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6.03</td>
<td>6.06</td>
<td>6.045</td>
</tr>
<tr>
<td>3</td>
<td>6.21</td>
<td>6.24</td>
<td>6.225</td>
</tr>
<tr>
<td>4</td>
<td>6.35</td>
<td>6.39</td>
<td>6.370</td>
</tr>
<tr>
<td>5</td>
<td>6.47</td>
<td>6.51</td>
<td>6.490</td>
</tr>
<tr>
<td>7</td>
<td>6.65</td>
<td>6.68</td>
<td>6.665</td>
</tr>
<tr>
<td>10</td>
<td>6.83</td>
<td>6.87</td>
<td>6.850</td>
</tr>
</tbody>
</table>

Bid/Offer rates differ by 3 to 4 basis points. Average of bid and offer rates is the swap rate.

\[ B_{fix} \equiv \text{Value of fixed-rate bond underlying the swap} \]
\[ B_{fl} \equiv \text{Value of floating-rate bond underlying the swap} \]

Swaps have zero value, so \( B_{fix} = B_{fl} \).
Comparative Advantage

Companies may have trouble accessing good fixed rate or floating rate loans, depending on their **credit rating**. Swaps allow for the company to convert fixed-rate loans from floating depending on its needs.

**Credit rating** describes likelihood of a company defaulting on a loan. S&P ratings go AAA, AA, A, BBB, BB, B, CCC. LIBOR is the rate at which AA-rated banks borrow for periods between 1 and 12 months.

**Example**: Consider two companies AAACo and BBBCo, both wish to borrow $10 million for 5 years and have been offered rates

<table>
<thead>
<tr>
<th></th>
<th>Fixed</th>
<th>Floating</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAACo</td>
<td>4.0%</td>
<td>6-month LIBOR +0.3%</td>
</tr>
<tr>
<td>BBBCo</td>
<td>5.2%</td>
<td>6-month LIBOR +1.0%</td>
</tr>
</tbody>
</table>

- BBBCo has a worse credit rating, so it borrows at a higher rate. Wants a fixed-rate loan.
- AAACo has a better credit rating, so it borrows at a cheaper rate. Wants a floating-rate loan.
- Difference in fixed-rate = 1.2% whereas difference in floating-rate = 0.7%.
- BBBCo seems to have a comparative advantage in floating-rate markets
- AAACo seems to have a comparative advantage in fixed-rate markets
- Difference leads to a swap negotiation
Comparative Advantage, cont.

• AAACo borrows fixed-rate funds at 4% per annum
• BBBCo borrows floating-rate funds at LIBOR plus 1% per annum
• Assume direct negotiation. AAACo agrees to pay BBBCo interest at 6-month LIBOR on $10 million. In return BBBCo agrees to pay AAACo interest at a fixed rate of 3.95% per annum on $10 million.

• AAACo
  – pays 4% per annum to outside lenders
  – receives 3.95% per annum from BBBCo
  – pays LIBOR to BBBCo

Net effect for AAACo is AAACo pays LIBOR plus 0.05% per annum or 0.25% less than direct lending.

• BBBCo
  – pays LIBOR plus 1% per annum to outside lenders
  – receives LIBOR from AAACo
  – pays 3.95% per annum to AAACo

Net effect for BBBCo is BBBCo pays 4.95% per annum. This is 0.25% less than direct lending.

• This swap is structured so that net gain is same for both parties. Here we take the difference of the fixed-rate = 1.2% and difference of floating rate = 0.7%. Then the difference between the two differences is 0.5%. The gain is split evenly.
• Suppose there is a financial intermediary: Then AAACo borrows at LIBOR + 0.07% and BBBCorp borrows at 4.97%. Financial company earns 0.04% per year.

• The gain for both borrowing companies is 0.23% per year. Total gain to all parties is 0.50% (including the financial intermediary.)
Swap Rates

Recall swap rate is the average of

- the fixed rate that a swap market maker is prepared to pay in exchange for receiving LIBOR (bid)
- the fixed rate that it is prepared to receive in return for paying LIBOR (offer)

Swap rates are nearly risk free.

A financial institute can earn the 5-year swap rate by

- Lending the principal for the first 6 months to a AA-borrower and then relend it for successive 6-month periods to other AA-borrowers
- Enter into a swap to exchange the LIBOR income for the 5-year swap rate.

In other words 8-year swap rate is an interest rate with credit risk corresponding to 16 consecutive 6-month LIBOR loans to AA companies.

Swap rates are less than AA borrowing rates since more attractive to lend money for short periods of time (6-month) than for long periods of time to retain liquidity.
Determining Swap/Zero Rates

- Use of LIBOR rates as risk-free rates used to price futures contracts.
- Only know LIBOR rates up to 12-month period.
- Extend LIBOR rates past 12 months via Eurodollar futures (up to 5 years, usually)
- Traders use swap rates to extend LIBOR zero curve further. Called the LIBOR/Swap zero curve.
- How to determine the curve:
  - New floating-rate bond is always equal to its principal value (par value) when using LIBOR/Swap zero curve for discounting. (Since rate of interest is LIBOR and LIBOR is discount rate)
  - Next $B_{fl} = B_{fix}$ for a new swap where fixed rate equals swap rate (again at start). This implies both $B_{fl}$ and $B_{fix}$ equal the notional principal.
  - Together imply that swap rates define a par yield bond.
  - Use bootstrap argument.
Determining Swap/Zero Rates

**Example:** The 1-year LIBOR rate is 10%. A bank trades swaps where a fixed rate of interest is exchanged for 12-month LIBOR with payments being exchanged annually. The 2-and 3-year swap rates (expressed with annual compounding) are 11% and 12% per annum. Estimate the 2- and 3-year LIBOR zero rates.

The two-year swap rate implies that a two-year LIBOR bond with a coupon of 11% sells for par. If $R_2$ is the two-year zero rate:

$$11e^{-0.10\times1.0} + 11e^{-R_2\times2.0} = 100$$

so that $R_2 = 0.1046$. The three-year swap rate implies that a three-year LIBOR bond with coupon of 12% sells for par. If $R_3$ is the three-year zero rate

$$12e^{-0.10\times1.0} + 12e^{-0.1046\times2.0} + 112e^{-R_2\times3.0} = 100$$

so that $R_3 = 0.1146$. The 2- and 3-year rates are 10.46% and 11.46% with continuous compounding.
Principal is not exchange in swap agreements.

In valuing swaps helpful to think of the principal as being exchanged. Then from the perspective of the floating-rate payer, a swap can be regarded as a long position in fixed rate bond and short position in floating rate bond. Yields a swap value of

\[ V_{swap} = B_{fix} - B_{fl} \]

where \( B_{fl} \) is the value of the floating-rate bond and \( B_{fix} \) is the value of the fixed-rate bond.

From the perspective of the fixed-rate payer, a swap can be regarded as a short position in fixed rate bond and long position in floating rate bond. Yields a swap value of

\[ V_{swap} = B_{fl} - B_{fix} \]

Note that a bond is worth the notional interest immediately after interest payment, since LIBOR has just been rolled-over and fair market value has been issued.
Valuation

If the notional principal is $L$, the next exchange of payments is at time $t^*$, and the floating payment will be made at time $t^*$ - determined at the last payment date - is $k^*$.

Immediately after payment $B_{fl} = L$. Therefore, immediately before the payment

$$B_{fl} = L + k^*$$

The floating-rate bond can be regarded as providing a single cash flow of $L + k^*$ at time $t^*$. Discounting, the value today is

$$(L + k^*) e^{-r^*t^*}$$

where $r^*$ is the LIBOR/swap zero rate for maturity of $t^*$. 
Example:

Suppose a financial institution has agreed to pay 6-month LIBOR and receive 8% per annum with semiannual compounding on a notional principal of $100 million. The swap has a remaining life of 1.25 years. The LIBOR rates with continuous compounding for 3-month, 9-month, and 15-month maturities are 10%, 10.5%, and 11%, respectively. The 6-month LIBOR rate at the last payment date was 10.2% with semiannual compounding.

The fixed-rate bond has cash flows of 4, 4 and 104 on the payment dates. Discounting yields discount factors

\[
e^{-0.10 \times 0.25} = 0.9753 \quad e^{-0.105 \times 0.75} = 0.9243 \quad e^{-0.11 \times 1.25} = 0.8715
\]

Therefore, we can calculate the present value of the bond as

\[
4 \times 0.9753 + 4 \times 0.9243 + 104 \times 0.8715 = $98.238 \text{ million}
\]
Computing the value of the floating-rate bond, we have $k^* = 0.5 \times 0.102 \times 100 = \$5.1$ million and $t^* = 0.25$. The floating-rate bond can be valued as though it produces a cash flow of $\$105.1$ million in 3 months.

<table>
<thead>
<tr>
<th>Time</th>
<th>$B_{fix}$ cash flow</th>
<th>$B_{fl}$ cash flow</th>
<th>Discount factor</th>
<th>Present value $B_{fix}$ cash flow</th>
<th>Present value $B_{fl}$ cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>4.00</td>
<td>105.10</td>
<td>0.9753</td>
<td>3.901</td>
<td>102.505</td>
</tr>
<tr>
<td>0.75</td>
<td>4.00</td>
<td>0.9243</td>
<td>3.697</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td>104.00</td>
<td>0.8715</td>
<td>90.640</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total:</td>
<td></td>
<td></td>
<td>98.238</td>
<td>102.505</td>
<td></td>
</tr>
</tbody>
</table>

Value of the swap is $B_{fix} - B_{fl} = 98.238 - 102.505 = \$ - 4.267$ million.
A swap can be characterized as a portfolio of FRA’s.

In the 3-year Microsoft-Intel swap, the first exchange of payments is known at the swap negotiations.

The next 5 exchanges can be regarded as forward rate agreements, i.e. the exchange on Sept. 5, 2005 is an FRA of 5% for interest at the 6-month rate observed March 5, 2005, etc.

Since FRA’s can be valued by assuming that forward interest rates are realized, then our plain vanilla swap can be valued via FRA valuing:

1. Use the LIBOR/swap zero curve to calculate forward rates for each of the LIBOR rates that will determine swap cash flows
2. Calculate swap cash flows on the assumption that the LIBOR rates will equal the forward rates
3. Discount these swap cash flows using the LIBOR/swap zero curve to obtain the swap value
Example:

Consider our 1.25 year swap example. Recall:

Our company pays 6-month LIBOR and receives 8% per annum on semiannual compounding on a notional principal of $100 million.

Swap has 1.25 years remaining. LIBOR rates with continuous compounding are 3-month, 9-month, 15-month maturities at 10%, 10.5%, and 11%, respectively. 6-month LIBOR rate at last payment was 10.2%.

Cash flow to be exchanged in 3 months:

- Fixed rate of 8% generates cash of $100 \times 0.08 \times 0.5 = 4$ million.
- Floating rate of 10.2%, set 3 months ago lead to a payment of $100 \times 0.102 \times 0.5 = 5.1$ million.
- Net cash flow is $4 - 5.1 = -1.1$ million.
- Discounting yields $-1.1 \times e^{-0.10 \times 0.25} = -1.1 \times 9753 = -1.073$ million.

This is the present value.
In 9 months we have identical $4 million cash inflow. To compute the outflow we need to calculate the forward rate for the time between 3 and 9 months.

- Recall:
  \[ R_F = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1} = \frac{0.105 \times 0.75 - 0.10 \times 0.25}{0.5} = 0.1075 \]
  with continuous compounding.

- But if \( e^{r_c} = \left(1 + \frac{r_2}{2}\right)^2 \) implies
  \[ r_2 = 2 \left(e^{\frac{r_c}{2}} - 1\right) = 2 \left(e^{\frac{0.1075}{2}} - 1\right) = 0.11044 \]
  semiannual compounding rate.

- Cash outflow is \( 100 \times 0.11044 \times 0.5 = 5.522 \) million. Next cash outflow is \( 4 - 5.522 = -1.522 \).

- Discounting at the 10.5% rate yields \( -1.522e^{-0.105 \times 0.75} = -1.522 \times 0.9243 = -1.407 \).

This is the present value of the next exchange.
In 15 months we have identical $4 million cash inflow. To compute the outflow we need to calculate the forward rate for the time between 9 and 15 months.

- Recall:
  \[ R_F = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1} = \frac{0.11 \times 1.25 - 0.105 \times 0.75}{0.5} = 0.1175 \]
  with continuous compounding.

- But if \( e^{rc} = \left(1 + \frac{r_2}{2}\right)^2 \) implies
  \[ r_2 = 2 \left( e^{\frac{rc}{2}} - 1 \right) = 2 \left( e^{\frac{0.1175}{2}} - 1 \right) = 0.1210 \]
  semiannual compounding rate.

- Cash outflow is \( 100 \times 0.1210 \times 0.5 = 6.051 \) million. Next cash outflow is \( 4 - 6.051 = -2.051 \).

- Discounting at the 10.5% rate yields \( -2.051 e^{-0.11 \times 1.25} = -2.051 \times 0.8715 = -1.787 \).

This is the present value of the next exchange.
Example, cont.

In summary we have

<table>
<thead>
<tr>
<th>Time</th>
<th>Fixed cash flow</th>
<th>Floating cash flow</th>
<th>Net cash flow</th>
<th>Discount factor</th>
<th>Present value of net cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>4.0</td>
<td>-5.100</td>
<td>-1.100</td>
<td>0.9753</td>
<td>-1.073</td>
</tr>
<tr>
<td>0.75</td>
<td>4.0</td>
<td>-5.522</td>
<td>-1.522</td>
<td>0.9243</td>
<td>-1.407</td>
</tr>
<tr>
<td>1.25</td>
<td>4.0</td>
<td>-6.051</td>
<td>-2.051</td>
<td>0.8715</td>
<td>-1.787</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>-4.267</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total value of the swap is the sum of the values of the FRA’s: hence $-4.267 million.

Remarks:

1. Fixed rate in an interest rate swap is chosen so that the swap is worth zero at the start.
   Therefore, the at the outset the sum of the values of the FRA’s underlying the swap is zero.
2. Not all FRA’s have zero value, only the sum is zero.
A currency swap is an agreement to exchange principal and interest payments in one currency for principal and interest payments in another.

- Requires the principal be specified in each of the two currencies.
- Principal amounts (usually approximately equivalent) are usually exchanged at the beginning and at the end of the swap.
- At the end of the swap, values may be very different.
Currency Swaps

• Consider 5-year fixed-for-fixed currency swap agreement between IBM and BP entered on Feb. 1, 2004.

• IBM pays a fixed rate of interest of 7% in sterling and receives a fixed rate of interest of 4% in dollars from BP.

• Interest rate payments are made once a year and principal amounts of $15 million and £10 million.

• At outset, IBM pays $15 million and receives £10 million.

• Each year during life of swap, IBM receives $0.04 \times 15 = 0.6$ million dollars and pays £0.07 \times 10 = 0.7 million pounds.

• At end, IBM pays £10 million and receives $15 million.

<table>
<thead>
<tr>
<th>Date</th>
<th>Dollar cash flow (millions)</th>
<th>Sterling cash flow (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>February 1, 2004</td>
<td>-15.00</td>
<td>+10.00</td>
</tr>
<tr>
<td>February 1, 2005</td>
<td>+0.60</td>
<td>-0.70</td>
</tr>
<tr>
<td>February 1, 2006</td>
<td>+0.60</td>
<td>-0.70</td>
</tr>
<tr>
<td>February 1, 2007</td>
<td>+0.60</td>
<td>-0.70</td>
</tr>
<tr>
<td>February 1, 2008</td>
<td>+0.60</td>
<td>-0.70</td>
</tr>
<tr>
<td>February 1, 2009</td>
<td>+15.60</td>
<td>-10.70</td>
</tr>
</tbody>
</table>
Suppose IBM has issued a $15 million bond at 4%, then the above swap converts the bond into a £10 million bond at 7%. (US dollar weakening?)

Suppose IBM has investments earning £10 million in UK markets earning 7%. The swap converts the investment into $15 million bond at 4%. (US dollar strengthening?)
Comparative Advantage?

Currency swaps can be initiated to take advantage of interest rate differences in foreign market rates.

• Consider General Motors and Qantas Airways both examine the 5-year fixed-rate borrowing costs and find:

<table>
<thead>
<tr>
<th></th>
<th>USD</th>
<th>AUD</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Motors</td>
<td>5.0%</td>
<td>12.6%</td>
</tr>
<tr>
<td>Qantas Airways</td>
<td>7.0%</td>
<td>13.0%</td>
</tr>
</tbody>
</table>

• Notes:
  – Australian rates higher than US rates
  – GM more credit worthy than Qantas, hence lower rates of borrowing
  – Spread in US market is 2% and spread in Australian market is 1.6%
  – Hence GM has a comparative advantage in US interest rate market and Qantas has a comparative advantage in the Australian market.

• Suppose GM wants to borrow 20 million AUD and Qantas wants to borrow 12 million USD. The currency exchange rate is 0.6000 USD per AUD.

• Ideal situations - GM can take advantage of advantage in US and Qantas can take advantage in Australia and use a swap.
Comparative Advantage, cont.

- Difference in US rates is 2% and difference in Australian rates is 0.4%, so the total expected gain is $2\% - 0.4\% = 1.6\%$ per annum.

- Use a financial intermediary. Therefore, GM
  - pays fixed-rate 5% USD to outside lender
  - earns 5% USD to intermediary
  - pays 11.9% AUD to intermediary
  Improved to 11.9% AUD from 12.6% AUD.

- Qantas
  - pays fixed-rate 13% AUD to outside lender
  - earns 13% AUD to intermediary
  - pays 6.3% USD to intermediary
  Improved to 6.3% USD from 7% USD.

- Financial Intermediary
  - earns 11.9% AUD from GM
  - pays 5% USD to GM
  - earns 6.3% USD from Qantas
  - pays 13.0% AUD to Qantas
  Earns 1.3% in USD and pays 1.1% in AUD. In particular a 2% per annum gain.
  In particular $0.013 \times 12 = 156,000$ USD is earned and $0.011 \times 20 = 220,000$ AUD.
  Convert to one or the other currency to get profit in USD or AUD.

- Can purchase forward contracts in AUD to hedge against risks.
Valuation of Currency Swaps

Fixed-for-fixed currency swaps can be valued by considering it as the difference between bonds or a portfolio of forward foreign exchange contracts.

**Valuation in Terms of Bond Prices:**

Let $V_{\text{swap}}$ denote the value in US dollars of an outstanding swap where dollars are received and foreign currency is paid.

$$V_{\text{swap}} = B_D - S_0 B_F$$

- $B_F$ is the value, measured in foreign currency, of the bond defined by the foreign cash flows on the swap.
- $B_D$ is the value of the bond defined by the domestic cash flows on the swap.
- $S_0$ is the spot exchange rate (expressed as number of dollars per unit of foreign currency).
- Value of swap is determined from
  - LIBOR rates in the two currencies.
  - Term structure of interest rates in the domestic currency.
  - Spot exchange rate.

Other hand have reverse value for foreign currency received and USD are paid is

$$V_{\text{swap}} = S_0 B_F - B_D$$
**Example:**

Suppose that the term structure of LIBOR/swap interest rates is flat in both Japan and the US.

- The Japanese rate is 4% per annum and the US rate is 9% per annum (both with continuous compounding).
- A financial institution has entered into a currency swap in which it receives 5% per annum in yen and pays 8% per annum in dollars once a year.
- The principals in the two currencies are $10 million and 1,200 million yen.
- The swap will last for another 3 years, and the current exchange rate is 110 yen per USD.

Valuing the first dollar income, we discount \(0.8 \times e^{-0.09 \times 1.0} = 0.7311\). On the other hand the yen payment has value of \(60 \times e^{-0.04 \times 1.0} = 57.65\).

Valuing the second dollar income, we discount \(0.8 \times e^{-0.09 \times 2.0} = 0.6628\). On the other hand the yen payment has value of \(60 \times e^{-0.04 \times 2.0} = 55.39\).

Valuing the third dollar income, we discount \(0.8 \times e^{-0.09 \times 3.0} = 0.6107\). On the other hand the yen payment has value of \(60 \times e^{-0.04 \times 3.0} = 53.22\).

Valuing the principal dollar income, we discount \(10.0 \times e^{-0.09 \times 3.0} = 7.6338\). On the other hand the yen payment has value of \(1200 \times e^{-0.04 \times 3.0} = 1064.30\).
## Valuation in Terms of Bond Prices

<table>
<thead>
<tr>
<th>Time</th>
<th>Cash flows on dollar bonds ($)</th>
<th>Present Value ($)</th>
<th>Cash flows on yen bond (yen)</th>
<th>Present value (yen)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8</td>
<td>0.7311</td>
<td>60</td>
<td>57.65</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
<td>0.6682</td>
<td>60</td>
<td>55.39</td>
</tr>
<tr>
<td>3</td>
<td>0.8</td>
<td>0.6107</td>
<td>60</td>
<td>53.22</td>
</tr>
<tr>
<td>3</td>
<td>10.0</td>
<td>7.6338</td>
<td>1200</td>
<td>1064.30</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>9.6439</td>
<td></td>
<td>1230.55</td>
</tr>
</tbody>
</table>

Therefore,

- The present value of the cash flows using the dollar discount rate of 9% is summed to give $9.6439 million = $B_D$.
- The present value of the cash flows using the yen discount rate of 4% is summed to give 1230.55 million yen = $B_F$.
- The value of the swap becomes:

\[
V_{swap} = B_D - S_0 B_F = 9.6439 - \frac{1}{110}1230.55 = 1.5430 \text{ million}
\]
Valuation in terms of Forward Contracts

Each exchange of payments in a fixed-for-fixed currency swap is a forward contract.

- LIBOR/swap term structure of interest rates is flat in both Japan and US.
- The Japanese rate is 4% per annum and US rate is 9% per annum (with continuous compounding).
- A financial institution has entered into a currency swap in which it receives 5% per annum in yen and pays 8% per annum in dollars once a year.
- Principals in the two currencies are $10 million and 1200 million yen. The swap will last for another 3 years and the current exchange rate is 110 yen for a US dollar.

Valuation. Financial institution pays $0.08 \times 10 = 0.8 \text{ million dollars}$ and receives $1200 \times 0.05 = 60 \text{ million yen}$ each year. Dollar principal of $10 \text{ million}$ is paid and the yen principal of $1200$ is received at the end of year 3.
Valuation in terms of Forward Contracts, cont.

Current spot rate is 0.009091 dollars per yen. When \( r = 4\% \) and \( r_f = 9\% \) so that the 1-year forward rate is

\[
F_0 = S_0 e^{(r_f-r)T} = 0.009091e^{(0.09-0.04)\times1} = 0.009557
\]

If the 1-year forward rate is realized, the yen cash flow in one year is worth \( 60 \times 0.009557 = 0.5734 \) million dollars. Net cash flow is \( 0.5734 - 0.8 = -0.2266 \) million.

Current spot rate is 0.009091 dollars per yen. When \( r = 4\% \) and \( r_f = 9\% \) so that the 2-year forward rate is

\[
F_0 = S_0 e^{(r_f-r)T} = 0.009091e^{(0.09-0.04)\times2} = 0.010047
\]

If the 2-year forward rate is realized, the yen cash flow in one year is worth \( 60 \times 0.010047 = 0.5734 \) million dollars. Net cash flow is \( 0.5734 - 0.8 = -0.2266 \) million.

Current spot rate is 0.009091 dollars per yen. When \( r = 4\% \) and \( r_f = 9\% \) so that the 3-year forward rate is

\[
F_0 = S_0 e^{(r_f-r)T} = 0.009091e^{(0.09-0.04)\times3} = 0.010562
\]

If the 3-year forward rate is realized, the yen cash flow in one year is worth \( 1260 \times 0.010562 = 13.30812 \) million dollars. Net cash flow is \( 13.30812 - 10.8 = 2.50812 \) million.
In total:

<table>
<thead>
<tr>
<th>Time</th>
<th>Dollar cash flow</th>
<th>Yen cash flow</th>
<th>Forward rate</th>
<th>Dollar value of yen cash flow</th>
<th>Net cash flow ($)</th>
<th>Present value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.8</td>
<td>60</td>
<td>0.009557</td>
<td>0.5734</td>
<td>-0.2266</td>
<td>-0.2071</td>
</tr>
<tr>
<td>2</td>
<td>-0.8</td>
<td>60</td>
<td>0.010047</td>
<td>0.6028</td>
<td>-0.1972</td>
<td>-0.1647</td>
</tr>
<tr>
<td>3</td>
<td>-0.8</td>
<td>60</td>
<td>0.010562</td>
<td>0.6337</td>
<td>-0.1663</td>
<td>-0.1269</td>
</tr>
<tr>
<td>3</td>
<td>-10.0</td>
<td>1200</td>
<td>0.010562</td>
<td>12.6746</td>
<td>+2.6746</td>
<td>2.0417</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.5430</td>
<td></td>
</tr>
</tbody>
</table>

Total value of the forward contracts is $1.5430 million.
Credit Risk

• Swap contracts are OTC agreements - private arrangements between two companies.

• Naturally entail credit risks

• Consider a financial company that serves as intermediary between two companies engaging in a swap

• If neither company defaults, the financial intermediary remains fully hedged (risk-free), since a decline in the value of one position is offset by the increase in the value of the opposite position.

• Suppose one side of the contract defaults. The intermediary still needs to honor the other side of the contract.
• Consider our original swap contract between Microsoft and Intel. After a short time, the contract with Microsoft has a positive value to the financial institution, whereas the contract with Intel has a negative value to the financial institution.

• Suppose Microsoft defaults on the swap, then the financial company would lose the positive value of this part of the option. The position is no longer hedged.

• To fix this the financial institution should find a third party willing to take the Microsoft position. In order to induce this, the financial company would have to pay the third party an amount equal to the value of the contract with Microsoft prior to default.
Credit Risk, cont.

- Market risk arises from possibility that market variables (interest rates/exchange rates) will move in a way to cause a contract to have negative value.

- Credit risk arises from the possibility of a default by the counterparty.

- Market risks are easier to hedge against than credit risks.
Other types of Swaps

Can have nonstandard payment schedules, instead of 6-month schedules.

- **Amortizing Swap** - a swap in which the principal decreases in a predetermined way. (The value decreases in time, i.e. loans or cars)
- **Step-up swap** - principal increases in a predetermined way. i.e. drawdowns on a loan agreements.
- **Forward swaps** - parties do not exchange interest rate payments until some future date (also called deferred swap)
- **Constant maturity swaps** - agreement to exchange a LIBOR rate for a swap rate. i.e. 6-month LIBOR applied to a certain principal for the 10-year swap rate applied to the same principal every 6 months for the next 5 years.
- **Constant maturity Treasury swaps** - similar. exchange LIBOR for particular Treasury rate
- **Compounding swaps** - interest on one or both sides is compounded forward to the end of the life of the swap according to preagreed rules. Only one payment date at the end of the life of the swap
- **LIBOR-in arrears swap** - the LIBOR rate observed on a payment date is used to calculate the payment on that date.
- **Accrual swap** - interest on one side of the swap accrues only when the floating reference rate is in a certain range.
Other Currency Swaps

Non fixed-for-fixed currency swaps.

- **Cross-currency interest rate swap** - a currency swap in which one side is floating rate (LIBOR) and the other side is a fixed rate in another currency.

- **Floating-for-floating currency swap** - both sides of the currency swap use LIBOR in respective currencies

- **Diff swap or quanto** - different floating rates used on the same principal amount in one of the currencies.
Other Swaps

- **Equity swap** - agreement to exchange the total return (dividends and capital gains) realized on an equity index for either a fixed or a floating rate of interest. i.e. total return on S&P 500 in successive 6-month periods might be exchanged for LIBOR, with both being applied to the same principal. Can be used to convert returns from a fixed or floating investments to returns in an equity index, for example.

- **Extendable swap** - one side has the option to extend the life of the swap beyond a specified period.

- **Puttable swap** - one side has the option to terminate the swap early.

- **Swaptions** - both extendable and puttable swaps are swaptions. (options on swaps)

- **Commodity swaps** - series of forward contracts on a commodity with different maturity dates and the same delivery prices.

- **Volatility swaps** - series of time periods. At the end of each period, one side pays an agreed volatility while the other side pays the historical volatility realized during the period. More about volatility soon!
Two most common types of swaps are plain vanilla interest rate swaps and currency swaps. In an interest rate swap, one party agrees to pay the other party interest at a fixed rate on a notional principal for a number of years. In return it receives interest at a floating rate on the same notional principal amount for the same period of time.

Principal amounts usually are not exchanged in an interest rate swaps. In a currency swap, principal amounts are usually exchanged at both the beginning and the end of the life of the swap.

In a currency swap one party agrees to pay interest on a principal amount in one currency. In return it receives interest on a principal amount in a different currency.

For the party paying interest in the foreign currency, the foreign principal is received, and the domestic principal is paid at the beginning of the life of the swap.

At the end of the swap, the foreign principal is paid and the domestic principal is received.

Interest rate swap can be used to transform floating-rate loan/asset into a fixed-rate loan/asset, or vice-versa.

Currency swap can be used to transform a loan/investment in one currency into a loan/investment in another currency.

Two ways of valuing interest rate and currency swaps. First method: swap is a long position in one bond and a short position in another bond. Second method: swap is a portfolio of forward contracts.
Options are different from Forward and Futures contracts in that an option gives the right to do something, but does not need to exercise this right.

Forward and Future contracts ensure that the parties commit to an agreement.

**Types of Options:**

- **Call option** - gives the owner the right to buy an asset by a certain date for a certain price
- **Put option** - gives the owner the right to sell an asset by a certain date for a certain price
- **Expiration date or maturity date** - is the date in which the call/put option expires.
- **Exercise price or strike price** - is the price specified in the option contract.
- **American option** - is an option that can be exercised any time up to expiration date.
- **European option** - is an option that can be exercised only at expiration date.
Call Options

Example:

- Investor believes eBay will gain value.
- Investor buys European call option with strike price of $100 to purchase 100 eBay shares.
- Suppose the current stock price is $98.
- Expiration date of option is in 4 months, and the price of an option to purchase one share is $5.
- Initial investment is $5 \times 100 = $500.
- If the stock price on the date is less than $100, the investor will not exercise, and loses $500.
- If the stock price is above $100 on the expiration date, the option will be exercised. Suppose the price is $110 then the investor buys 100 shares at $100 per share and sells immediately at $110. Thus each share earns $10 per share, or $1000. Net profit is $1000 - $500 = $500.
- In general we have the value for each contract:

\[
100 \times \max\{S_T - 100 + 5, -5\}
\]

for spot price $S_T$ at termination.
Put Options

Example:

• Consider an investor who believes IBM will lose value and buys a put option to sell 100 shares in IBM with a strike price of $70.
• Suppose the current stock price is $65 with expiration in 3 months and the price of an option to sell a single share of $7.
• Initial investment is $700. Only exercised if the stock price is below $70 on the expiration date.
• Suppose price is $55 then a gain of $15 per share or $1500. Net profit is 1500-700 = $800.
• If price above $77 then the option is not exercised.
• In general we have the value for each contract:

\[ 100 \times \max\{ -7 + 70 - S_T, -7 \} \]

for spot price \( S_T \) at termination.
Homework

Due Oct. 10, 5PM.

• 6.10, 6.13, 7.12

• Graded: 6.23, 6.25, 7.20, 7.21, 7.22