Credit Derivatives & Exotic Options
Credit Derivatives

Credit derivatives allow companies to trade credit risks in the same way that they trade market risks.

Companies can enter into credit derivative contracts to protect themselves from credit defaults of their lenders/borrowers.
The most popular credit derivative is a **credit default swap** (CDS)

- A CDS contract provides insurance against the risk of a default by a particular company.
- The company is known as the **reference entity** and a default by the company is known as a **credit event**.
- The buyer of the insurance obtains the right to sell bonds issued by the company for their face value when a credit event occurs.
- The sellers of the insurance agrees to buy the bonds for their face value when a credit event occurs.
- The total face value of the bonds that can be sold is known as the credit default swap’s **notional principal**.

**Figure 21.1** Credit default swap.

![Diagram of credit default swap](image)
Features of a CDS

- Buyers of a CDS makes periodic payments to the seller until the end of the life of the CDS or until a credit event occurs.
- The payments are typically made in arrears every quarter, every half year, or every year.
- The settlement, in the event of a default, involves either physical delivery of the bonds or a cash payment.
The regular quarterly, semiannual, or annual payments form the buyer of protection to the seller of protection cease when there is a credit event. However, because these payments are made in arrears, a final accrual payment by the buyer is usually required.

In the example a buyer would be required to pay to the seller the amount of the annual payment accrued between Mar. 1, 2007 and June 1, 2007

- The total amount paid per year, as a percent of the notional principal is known as the CDS spread.
- Many large bands are market makers in the credit default swap market.
- When quoting on a new 5-year CDS on Ford, a market maker might bid 250 basis points and offer 260 basis points.
Participants in credit derivatives markets have developed indices to track credit default swap spreads. In 2004, there were agreements between different producers of indices which led to some consolidation. Two such indices:

1. The 5- and 10-year CDX NA IG indices tracking the credit spread for 125 investment grade North American companies; and
2. The 5- and 10-year iTraxx Europe indices tracking the credit spread for 125 investment grade European companies

In addition to monitoring credit spreads, indices provide a way for market participants can easily buy or sell a portfolio of credit default.
Valuation of Credit Default Swaps

Mid-market CDS spreads on individual reference entities can be calculated from default probability estimates.

We go through a detailed example:

Suppose that the probability of a reference entity defaulting during a year conditional on no earlier default is 2%. Suppose we have the following survival probabilities as seen at time zero for each of the 5 years:

<table>
<thead>
<tr>
<th>Time in years</th>
<th>Default probability</th>
<th>Survival probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0200</td>
<td>0.9800</td>
</tr>
<tr>
<td>2</td>
<td>0.0196</td>
<td>0.9604</td>
</tr>
<tr>
<td>3</td>
<td>0.0192</td>
<td>0.9412</td>
</tr>
<tr>
<td>4</td>
<td>0.0188</td>
<td>0.9224</td>
</tr>
<tr>
<td>5</td>
<td>0.0184</td>
<td>0.9039</td>
</tr>
</tbody>
</table>
We have:

- The probability of default during the first year is 0.02 and the probability the reference entity will survive until the end of the first year is 0.98.
- The probability of a default during the second year is \(0.02 \times 0.98 = 0.0196\) and the probability of survival until the end of the second year is \(0.98 \times 0.98 = 0.9604\). The probability of default during the third year is \(0.02 \times 0.9604 = 0.0192\), etc.

We will assume that defaults always happen halfway through a year and that payments on the CDS are made once a year, at the end of each year.

We also assume that the risk-free LIBOR interest rate is 5% per annum with continuous compounding and the recovery rate is 40%.
We first calculate the present value of the payments made on the CDS assuming that payments are made at the rate of $s$ per year and the notional principal is $1. For example there is a 0.9412 probability that the third payment of $s$ is made. The expected payment is therefore $0.9412s$ and its present value is

$$0.9412se^{-0.05\times2.5} = 0.0102$$

The total present value of the expected payments is $4.0704s$.

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>Probability of survival</th>
<th>Expected payment</th>
<th>Discount factor</th>
<th>PV of expected payment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9800</td>
<td>0.9800s</td>
<td>0.9512</td>
<td>0.9322s</td>
</tr>
<tr>
<td>2</td>
<td>0.9604</td>
<td>0.9604s</td>
<td>0.9048</td>
<td>0.8690s</td>
</tr>
<tr>
<td>3</td>
<td>0.9412</td>
<td>0.9412s</td>
<td>0.8607</td>
<td>0.8101s</td>
</tr>
<tr>
<td>4</td>
<td>0.9224</td>
<td>0.9224s</td>
<td>0.8187</td>
<td>0.7552s</td>
</tr>
<tr>
<td>5</td>
<td>0.9039</td>
<td>0.9039s</td>
<td>0.7788</td>
<td>0.7040s</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
<td><strong>4.0704s</strong></td>
</tr>
</tbody>
</table>
• Next we calculate the expected present value of the payoff assuming a notional principal of $1. Assuming that defaults always happen halfway through a year. For example, there is a 0.0192 probability of a payoff halfway through the third year. Given that the recovery rate is 40% the expected payoff at this time is

\[ 0.0192 \times 0.6 \times 1 = 0.0115 \]

The present value of the expected payoff is

\[ 0.0115e^{-0.05 \times 2.5} = 0.0102 \]

The total present value of the expected payoffs is $0.0511.

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>Probability of default</th>
<th>Recovery rate</th>
<th>Expected payoff ($)</th>
<th>Discount factor</th>
<th>PV of expected payoff ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.0200</td>
<td>0.4</td>
<td>0.0120</td>
<td>0.9753</td>
<td>0.0117</td>
</tr>
<tr>
<td>1.5</td>
<td>0.0196</td>
<td>0.4</td>
<td>0.0118</td>
<td>0.9277</td>
<td>0.0109</td>
</tr>
<tr>
<td>2.5</td>
<td>0.0192</td>
<td>0.4</td>
<td>0.0115</td>
<td>0.8825</td>
<td>0.0102</td>
</tr>
<tr>
<td>3.5</td>
<td>0.0188</td>
<td>0.4</td>
<td>0.0113</td>
<td>0.8395</td>
<td>0.0095</td>
</tr>
<tr>
<td>4.5</td>
<td>0.0184</td>
<td>0.4</td>
<td>0.0111</td>
<td>0.7985</td>
<td>0.0088</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0511</td>
</tr>
</tbody>
</table>
The final step entails computing the accrual payment made in the event of a default. For example, there is a 0.0192 probability that there will be a final accrual payment halfway through the third year. The accrual payment is $0.5s$. The expected accrual payment at this time is therefore

$$0.0192 \times 0.5s = 0.0096s$$

Its present value is

$$0.0096se^{-0.05\times 2.5} = 0.0085s$$

The total present value of the expected accrual payments is $0.0426s$.

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>Probability of default</th>
<th>Expected accrual payment</th>
<th>Discount factor</th>
<th>PV of expected accrual payment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.0200</td>
<td>0.0100s</td>
<td>0.9753</td>
<td>0.0097s</td>
</tr>
<tr>
<td>1.5</td>
<td>0.0196</td>
<td>0.0098s</td>
<td>0.9277</td>
<td>0.0091s</td>
</tr>
<tr>
<td>2.5</td>
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<td>0.8825</td>
<td>0.0085s</td>
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<td>0.0094s</td>
<td>0.8395</td>
<td>0.0079s</td>
</tr>
<tr>
<td>4.5</td>
<td>0.0184</td>
<td>0.0092s</td>
<td>0.7985</td>
<td>0.0074s</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td>0.0426s</td>
</tr>
</tbody>
</table>
Therefore, the expected payments is

\[ 4.0704s + 0.0426s = 4.1130s \]

From our second table the present value of the expected payoff is 0.0511. Equating the two, we obtain the CDS spread for a new CDS is

\[ 4.1130s = 0.0511 \]

or \( s = 0.0124 \). The mid-market spread should be 0.0124 times the principal or 124 basis points per year.

In practice we are likely to find that calculations are more extensive than those in Table 21.2.
Estimating Default Probabilities

The default probabilities used to value a CDS should be risk-neutral default probabilities, not real-world default probabilities. Risk-neutral default probabilities can be estimated from bond prices or asset swaps as earlier.
Collateralized Debt Obligations

A Collateralized Debt Obligations (CDO) is a way of creating securities with widely different risk characteristics from a portfolio of debt instruments.

Ingredients of the current market crisis.

Consider the following example: Four types of tranches (securities) are created from a portfolio of bonds (or mortgages).

- The first tranche has 5% of the total bond principal and absorbs all credit losses from the portfolio during the life of the CDO until they reached 5% of the total bond principal.
- The second tranche has 10% of the total bond principal and absorbs all losses during the life of the CDO in excess of 5% of the principal up to a maximum of 15% of the principal.
- The third tranche has 10% of the total bond principal and absorbs all losses during the life of the CDO in excess of 15% of the principal up to a maximum of 25% of the principal.
- The fourth tranche has 75% of the principal absorbs all losses in excess of 25% of the principal.
The yields are the rates of interest paid to tranche holders.
These rates are paid on the balance of the principal remaining in the tranche after losses have been paid.

- For the first tranche, initially a return of 35% is paid on the whole amount invested by tranche 1 holders.
- After losses equal to 1% of the total bond principal have been experienced, tranche 1 holders have lost 20% of their investment and the return is paid on only 80% of the original amount invested.
- Tranche 1 is referred to as the **equity tranche**.
- A default loss of 2.5% on the bond portfolio translates into a loss of 50% of the tranche’s principal.
- Tranche 4 by contrast is usually given an Aaa rating. Defaults on the bond portfolio must exceed 25% before the holders of this tranche are responsible for any credit losses.
- The creator of the CDO normally retains the equity tranche and sells the remaining tranches in the market.
- A CDO provides a way of creating high quality debt from average quality or low quality debt.
Single Tranche Trading

The portfolios of 125 companies are used to generate the CDX and iTraxx indices. The market uses the portfolios to define standard CDO tranches.

The trading of these standard tranches is known as single tranche trading.

• A single tranche trade is an agreement where one side agrees to sell protection against the losses on a tranche and the other side agrees to buy the protection.
• The tranche is not part of a synthetic CO buy cash flows are calculated in the same way as if it were part of a synthetic CDO.
• The tranche is referred to as unfunded because it has not been created by selling credit default swaps or buying bonds.
• In the case of the CDX index, the equity tranche covers losses between 0% and 3% of the principal. The second tranche, which is referred to as the mezzanine tranche covers losses between 3% and 7%. The remaining tranches cover losses from 7% to 10%, 10% to 15%, 15% to 30%.
• In the case of iTraxx Europe index, the equity tranche covers losses between 0% and 3%, the mezzanine tranche covers losses between 3% and 6%. The remaining tranches cover losses from 6% to 9%, 9% to 12%, 12% to 22%.
Example: Consider the mid-market quotes for 5-year CDX and iTraxx tranches on August 4, 2004.

<table>
<thead>
<tr>
<th>CDX IG NA</th>
<th>Tranche</th>
<th>0%–3%</th>
<th>3%–7%</th>
<th>7%–10%</th>
<th>10%–15%</th>
<th>15%–30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quote</td>
<td>41.8%</td>
<td>347</td>
<td>135.5</td>
<td>47.5</td>
<td>14.5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>iTraxx Europe</th>
<th>Tranche</th>
<th>0%–3%</th>
<th>3%–6%</th>
<th>6%–9%</th>
<th>9%–12%</th>
<th>12%–22%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quote</td>
<td>27.6%</td>
<td>168</td>
<td>70</td>
<td>43</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

On that date the CDX index level was 63.25 basis points and the iTraxx index was 42 basis points.

- The mid-market price of mezzanine protection for the CDX IG NA was 347 basis points per year while that for iTraxx Europe was 168 basis points per year.
- Equity tranche is quoted differently from the other tranches.
- The market quote of 41.75% for CDX means that the protection seller receives an initial payment of 41.75% of the principal plus a spread of 500 basis points per year.
- Similarly the market quote of 27.6% for iTraxx mean that the protection seller receives an initial payment of 27.6% of the principal plus a spread of 500 basis points per year.
A **convertible bond** is a bond issued by a company in which the holder has the option to exchange the bonds for the company’s stock at certain times in the future.

- The **conversion factor** is the number of shares of stock obtained for one bond.
- The bonds are almost always callable (i.e. the issuer has the right to buy them back at certain times at a predetermined price).
- The holder always has the right to convert the bond once it has been called.
- The call feature is usually a way of forcing conversion earlier than the holder would otherwise choose. Sometimes the holder’s call option is conditional on the price of the company’s stock being above a certain level.
- Credit risk plays an important role in the valuation of convertibles. If we ignore credit risk, we will get poor prices because the coupons and principal payments on the bond will be overvalued.
Pricing a convertible:

- Assume that the issuer’s stock follows a Wiener process except that there is a probability $\lambda \Delta t$ that there will be a default in each short period of time $\Delta t$.
- In the event of a default the stock price drops to zero and there is a recovery on the bond. The variable $\lambda$ is the risk-neutral default intensity.
- We can represent the stock price process by varying the usual binomial tree so that at each node
  1. A probability $p_u$ of a percentage up movement of size $u$ over the next time period of length $\Delta t$.
  2. A probability $p_d$ of a percentage down movement of size $d$ over the next time period of length $\Delta t$.
  3. A probability $1 - e^{-\lambda \Delta t} \approx \lambda \Delta t$ that there will be a default with the stock price moving to zero over the next time period of length $\Delta t$.

Parameter values are chosen, as before, to match the first two moments of the stock price distribution, yield:

\[
p_u = \frac{a - de^{-\lambda \Delta t}}{u - d}, \quad p_d = \frac{ue^{-\lambda \Delta t} - a}{u - d}, \quad u = e^{\sqrt{\frac{\sigma^2}{\Delta t} - \lambda \Delta t}}, \quad d = \frac{1}{u}
\]

and $a = e^{(r-q)\Delta t}$, $r$ is the risk-free rate and $q$ is the dividend yield on the stock.
• The life of the tree is set to be equal to the life of the convertible bond.
• The value of the convertible bond at the final nodes of the tree is calculated based on any conversion options that the holder has at that time. We then work back through the tree.
• At nodes where the terms of the instrument allow conversion, we test whether conversion is optimal. We also test whether the position of the issuer can be improved by calling the bonds.
• If so we assume that the bonds are called and retest whether conversion is optimal. This is equivalent to setting the value at the node equal to

$$\max \left[ \min(Q_1, Q_2), Q_3 \right]$$

where $Q_1$ is the value given by the rollback (assuming that the bond is neither converted nor called at the node). $Q_2$ is the call price. $Q_3$ is the value if conversion takes place.
Exotic Options

American and European options are known as **plain vanilla options**. Recently there have developed nonstandard **exotic options** that have become parts of portfolios.

May be useful

- for hedging purposes
- for tax, accounting, legal, or regulatory reasons
- since they are designed to reflect a view on potential future movements in particular market variables

Throughout we assume the asset provides a known yield of $q$. 
A **package** is a portfolio consisting of standard European calls, standard European puts, forward contracts, cash, and the underlying asset itself.

Examples of such packages include the bull spreads, bear spreads, butterfly spreads, calendar spreads, straddles, etc.

Often the package is structured so as to have zero cost initially.
In practice American options traded in OTC markets have nonstandard features.

1. Early exercise may be restricted to certain dates. The instrument is then known as a **Bermudan option**.
2. Early exercise may be allowed during only part of the life of the option. For example, there may be an initial "lock-out" period with no early exercise.
3. The strike price may change during the life of the option.

Nonstandard American options can usually be valued using a binomial tree. At each node the test for early exercise is adjusted to reflect the terms of the option.
A **forward start option** are options that will start at some time in the future.

Executive stock options can be viewed as a type of forward start option. This is because a company commits to granting at-the-money options to employees in the future.

- Consider a forward start at-the-money European call option that will start at time $T_1$ and mature at time $T_2$.
- Suppose that the asset price is $S_0$ at time zero and the $S_1$ at time $T_1$.
- To value the option, we note from the European option pricing formulas that the value of an at-the-money call option is proportional to the asset price, since

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2) = S_0 \left( N(d_1) - e^{-rT} N(d_2) \right)$$

where $d_1 = \ln \frac{S_0}{K} + (r + \frac{\sigma^2}{2})T = (r + \frac{\sigma^2}{2})T$ and $d_2 = (r - \frac{\sigma^2}{2})T$.

The value of the forward start option at time $T_1$ is therefore $cS_1/S_0$, where $c$ is the value at time zero of an at-the-money option that lasts for $T_2 - T_1$. 


Using risk-neutral valuation the value of the forward start option at time zero is

\[ e^{-rT_1} \hat{E} \left[ \frac{cS_1}{S_0} \right] \]

where \( \hat{E} \) is the expected value in the risk-neutral world.

Since \( c \) and \( S_0 \) are known and \( \hat{E} [S_1] = S_0 e^{(r-q)T_1} \), then the value of the forward start option is \( ce^{-qT_1} \). For a non-dividend-paying stock, \( q = 0 \) and the value of the forward start option is exactly the same as the value of a regular at-the-money option with the same life as the forward start option.
Compound Options

**Compound options** are options on options. There are four main kinds

- call on a call
- put on a call
- call on a put
- put on a put

We will price these options next week.
A chooser option has the feature that after a specified period of time, the holder of the option can chose whether the option is a call or a put.

Suppose that the time when the choice is made is $T_1$. The value of the chooser option at this time is

$$\max\{c, p\}$$

where $c$ is the value of the call underlying the option and $p$ is the value of the put underlying the option.

If the options underlying the chooser option are both European and have the same strike price, put-call parity can be used to provide a valuation formula.

Suppose that $S_1$ is the asset price at time $T_1$, $K$ is the strike price, $T_2$ is the maturity of the options, and $r$ is the risk-free interest rate. Put-call parity implies

$$\max\{c, p\} = \max\{c, c + Ke^{-r(T_2-T_1)} - S_1e^{-q(T_2-T_1)}\}$$

$$= c + e^{-q(T_2-T_1)} \max\{0, Ke^{-(r-q)(T_2-T_1)} - S_1\}$$
Hence the chooser is a sum of

1. A call option with strike price $K$ and maturity $T_2$
2. $e^{-q(T_2 - T_1)}$ put options with strike price $Ke^{-(r-q)(T_2 - T_1)}$ and maturity $T_1$.

We can use Black-Scholes to compute the exact values.
Barrier Options

Barrier options are options where the payoff depends on whether the underlying asset’s price reaches a certain level during a certain period of time.

- A number of different types of barrier options regularly trade in the OTC market. They are attractive to some market participants because they are less expensive than the corresponding regular options.
- Barrier options can be classified as one of two types:
  - A **knock-out option** ceases to exist when the underlying asset price reaches a certain barrier.
  - A **knock-in option** comes into existence only when the underlying asset price reaches a barrier.

We will price some of these options next week.
1. What is the payoff for a short position in a put option where $S_T$ is the stock price and $K$ is the strike price, assuming no transaction cost? (circle one)
   (a) $\max\{K - S_T, 0\}$
   (b) $\max\{S_T - K, 0\}$
   (c) $\min\{K - S_T, 0\}$
   (d) $\min\{S_T - K, 0\}$

2. Which of the following cannot be calculated directly from a binomial tree (circle two) (circle two)
   (a) vega
   (b) delta
   (c) rho
   (d) gamma
   (e) theta
3. Which of the following is always positively related to the price of a European call option on a stock (circle two)

(a) The stock price
(b) The strike price
(c) The time to expiration
(d) The volatility
(e) The magnitude of dividends anticipated during the life of the option

4. Suppose that the price of gold at close of trading yesterday was $300 and its volatility was estimated as 1.3% per day. The price at the close of trading today is $298. Update the volatility estimate using

(a) The EWMA model with $\lambda = 0.94$
(b) The GARCH (1,1) model with $\omega = 0.000002$, $\alpha = 0.04$, and $\beta = 0.94$.

\[
\sigma_n^2 = 0.94\sigma_{n-1}^2 + (1 - 0.94)u_n^2
\]
\[
= 0.94 \times 0.013^2 + 0.06 \times \left(\frac{298 - 300}{300}\right)^2
\]
\[
= 0.01271^2
\]
(b)

\[ \sigma_n^2 = \omega + 0.94\sigma_{n-1}^2 + 0.04u_n^2 \]

\[ = 0.000002 + 0.94 \times 0.013^2 + 0.04 \times \left( \frac{298 - 300}{300} \right)^2 \]

\[ = 0.01275^2 \]

5. Sketch the equity volatility smile over the lognormal plot.
   (a) Explain the difference between the two curves
   (b) Give a reason for the difference

- Fear of a crash. Traders are concerned about the possibility of a crash, so they price the option accordingly.
- Leverage. As a company's equity declines in value, the equity becomes more risky and its volatility increases. As a company's equity increases in value, the equity becomes less risky and its volatility decreases.
6. Consider a position consisting of a $300,000 investment in gold and a $500,000 investment in silver. Suppose that the daily volatilities of these two assets are 1.8% and 1.2%, respectively, and that the coefficient of correlation between their returns is 0.6. What is the 10-day 97.5% VaR for the portfolio? How much does diversification reduce the VaR?

Let $X$ denote the gold and $Y$ denote the silver. Then

$$\sigma_X = 0.018 \times 300,000 = 5400 \quad \sigma_Y = 0.012 \times 500,000 = 6000$$

Compute the joint variance

$$\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2 + 2 \rho_{XY} \sigma_X \sigma_Y}$$

$$= \sqrt{5400^2 + 6000^2 + 2 \times 0.6 \times 5400 \times 6000}$$

$$= 10,200$$

The 10-day 97.5% VaR is

$$\sqrt{10} \times N^{-1}(2.5) \times 10,200 = \sqrt{10} \times 1.96 \times 10,200 = 63,220.25$$

The 10-day 97.5% VaR for gold and silver are

$$\sqrt{10} \times 1.96 \times 5400 = 33,409.55 \quad \sqrt{10} \times 1.96 \times 6000 = 37,188.39$$
The diversification reduces the VaR by

\[ 33,409.55 + 37,188.39 - 63,220.25 = 7437.68 \]

7. Consider a 6-month put option on a stock when the futures price is 18, the strike price is 17, risk–free rate is 5%, and volatility is 55%. Find the value of the option using a two–step binomial tree if the option is

(a) European
(b) American. Is there ever early exercise? When?

By hand.
8. Design a Monte Carlo method to compute the price of a European put option. Suppose the stock price is $S_0$, the strike price is $K$, the volatility is $\sigma$, the expected growth rate is $\mu$, and the risk-free interest rate is $r$. Explain all necessary steps and what exactly is computed at each step.

(a) For the $i$th iteration generate a random number from the standard normal distribution via

$$\epsilon_i = N^{-1}(RAN)$$

where $RAN$ produces a random number between 0 and 1 from a uniform distribution.

(b) Generate a stock value for the $i$th at the end time via

$$S_i(T) = S_0 \exp \left[ (\mu + \frac{\sigma^2}{2})T + \epsilon_i \sigma \sqrt{T} \right]$$

where $\mu, \sigma, T$ are fixed values.

(c) Evaluate the option value for the $i$th iteration

$$f_i = e^{-rT} \max \{ K - S_i(T), 0 \}$$

(d) Repeat steps (a) - (c) $N$ times and average to get the Monte-Carlo price.
9. Suppose that a futures price is $20, the strike price of a nine-month European call on the futures is $19, the risk-free rate is 8% per annum, and the value of the option is $3. What is the value of a put option with the same strike price and time to maturity?

Use the put-call parity for futures options:

\[ c + K e^{-rT} = p + S_0 e^{-rT} \]

or

\[ p = 3 + (19 - 20) e^{-0.08 \times 0.75} = 2.06 \]
10. Consider the following hedging scheme. You are running a large fund and wish to write earn
$200,000 from writing 1000 European call option contracts (each contract for 100 shares of
stock) at $2 per share. The stock price is currently $30 and the strike price for the call option
is $32. The risk-free interest rate is 5%, the volatility of the stock is 20% and the time to
maturity is 6 months.
In order to build a Black-Scholes Delta hedge, how many shares of the stock should you buy?
How much does it cost to buy these shares?

We use the Black-Scholes formula for the Delta: \( \Delta = N(d_1) \). Therefore,

\[
d_1 = \frac{\ln \frac{S_0}{K} + (r + \frac{\sigma^2}{2})T}{\sigma \sqrt{T}} = \frac{\ln \frac{30}{32} + (0.05 + \frac{0.2^2}{2})0.5}{0.2\sqrt{0.5}} = -0.209
\]

Then from the chart \( N(-0.209) = 0.4168 \). This implies we need

\[
0.4168 \times 100,000 = 41,680
\]

shares at a cost of

\[
41,680 \times 30 = $1,250,400
\]
Homework

Homework: Due April 2, 5PM.
Graded: 20.26, 21.28