Math 5525 Homework Assignment 3 Spring 2013

due April 8, 2013, at the beginning of class, 2:30pm

1. Give an example of two 2×2 matrices A, B such that

$$e^{A+B} \neq e^A e^B \,.$$

A proof that for your example the two sides are indeed different should be a part of the solution.

2. Exercise 1, page 115

3. Exercise 2, page 115

4. For all the three matrices A_1, A_2, A_3 in Problem 2 find their Jordan canonical form.

5*. (Optional) Calculate e^{tA_j} $j = 1, 2, 3$ for all the three matrices in Problem 2.

6*. (Optional) Let A be a complex $n \times n$ matrix such that all its eigenvalues have a strictly positive real part. Prove that

$$\int_0^\infty e^{-tA}\, dt = A^{-1} \,.$$

Remark: Here we integrate a matrix-values function of t. By definition, the identity $\int_{t_1}^{t_2} M(t)\, dt = B$ for some smooth matrix valued-function $M(t)$ and a matrix B of the same size simply means that $\int_{t_1}^{t_2} m_{ij}(t)\, dt = b_{ij}$ for all indices i, j.