Math 5525 Homework Assignment 4 Spring 2013

due April 29, 2013, at the beginning of class, 2:30pm

1. Consider the second order equation with (smooth) variable coefficients

$$x'' + p(t)x' + q(t)x = 0 \tag{1}$$

in an interval $I = (t_1, t_2)$. Assume that x_1, x_2 are two solutions of (1) and set

$$w = x_1 x_2' - x_2 x_1'. \tag{2}$$

The function w is called the *Wronskian* of the two solutions x_1, x_2. Show that

(i) w satisfies the equation $w' + p(t)w = 0$;

(ii) if $w(t_0) \neq 0$ for some $t_0 \in I$, then $w(t) \neq 0$ for every $t \in I$.

2. Find the general solution of the equation

$$x'' + \frac{x'}{t} = 1 \tag{3}$$

on the interval $(0, \infty)$. (Hint: First solve the homogeneous equation and then either use the variation of constants or try to guess a particular solution.)

3. Consider the equation

$$x'' + \frac{2x'}{t} + x = 0 \tag{4}$$

in $(0, \infty)$. Show that the substitution $x = \frac{y}{t}$ changes our equation with variable coefficients to an equation with constant coefficients and find a basis of the space of solutions of (4).

4.* (Optional) Exercises 1-3 on page 190.

5.* (Optional) For an $\mathbf{R}^2-$valued function $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ consider the system

$$x'' + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} x' + x = 0\,. \tag{5}$$

Find all bounded solutions of the system in the interval $(0, \infty)$.

Hint: one can either write our equation as a first order 4×4 system (by setting $x' = y$ and writing down the equation satisfied by by $z = \begin{pmatrix} x \\ y \end{pmatrix}$) or one can search directly for solutions of the form $be^{\lambda t}$ in a way similar to what we have done for first-order systems.