Problem 1.
Find the eigenvalues and the eigenfunctions for the Dirichlet and Neumann problems for the Laplacian on a rectangle \((0, a) \times (0, b)\).

Problem 2.
Prove that the wave equation
\[
\partial_t^2 u(t, x) = c^2 \Delta x u(t, x), \quad t > 0, \quad x \in \Omega \in \mathbb{R}^d
\]
with Dirichlet boundary conditions
\[
u(t, x) = 0 \quad \text{for} \quad x \in \partial \Omega, \quad t > 0,
\]
has a solution
\[
u(x, t) = \sum_n \left( A_n \cos(\sqrt{\lambda_n} c t) + B_n \cos(\sqrt{\lambda_n} c t) \right) v_n(x)
\]
where \(\lambda_n\) and \(v_n\) are, respectively, eigenvalues and eigenfunctions of the Dirichlet problem for the Laplacian in \(\Omega\).

Write in an analogous form the solution to the heat equation
\[
\partial_t^2 u(t, x) = c \Delta x u(t, x), \quad t > 0, \quad x \in \Omega \in \mathbb{R}^d
\]
with Dirichlet boundary conditions
\[
u(t, x) = 0 \quad \text{for} \quad x \in \partial \Omega, \quad t > 0.
\]

The rest: 6.3.9, 6.3.10, 6.3.18, 6.3.21, 6.3.23, 6.3.27, 6.3.31 from the usual textbook

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