

# Normal and Lognormal

## Normal

Normally distributed random variables  $X$  have expected value  $\mu$  and standard deviation  $\sigma$ .

## Lognormal

$Y$  is a lognormally distributed random variable if  $\ln(Y) = X$  is normally distributed. Write out the following using  $\mu = E(X)$  and  $\sigma^2 = \text{Var}(X)$ :

- $E(Y) =$
- $\text{Var}(Y) =$

Graph the pdf of a normally distributed random variable on the left, and the pdf of the corresponding lognormally distributed random variable on the right:

$X =$  (in terms of  $Y$ )       $Y =$  (in terms of  $X$ )

# Normal and Lognormal

## Normal

For an initial stock price of \$10, with increases/decreases of \$3 per step, draw an additive binomial tree with 4 steps below:

## Lognormal

For an initial stock price of \$10, with increases/decreases of 30% per step, draw a multiplicative binomial tree with 4 steps below:

Which tree has a price distribution symmetric about the mean price after 4 steps? Which tree has only positive prices at the end?

# Normal and Lognormal

## Normal

Write out cdf and pdf of  $Z \sim N(0,1)$ , using  $\Phi$  (capital) and  $\phi$  (lowercase) notation:

Formula for standardizing  $X \sim N(\mu, \sigma^2)$  to  $Z \sim N(0,1)$ :

$$Z =$$

Thus

$$P(X < x) = .$$

## Lognormal

For  $Y$  lognormally distributed,  $\ln(Y) = X$  normally distributed, write out pdf and cdf for  $Y$ :

By standardizing, write the following in terms of the function  $\Phi$ :

$$P(Y < y) = .$$