**Math 1001**                                      Name: _____

**Fall 2007**

**Test 2**                                         Student ID: _____

**11/7/07**

**Time allowed: 50 minutes**                       Section: 10:10 11:15 12:20

This exam includes 7 pages, including this one and a sheet for scratch work. There are a total of 6 questions on the exam, each with multiple parts.

**This test is closed book.** You are not allowed to consult the text or any notes you have. Scientific calculators are allowed, but not graphing calculators or any other calculators with more functions.

**Show your work.** Except where specifically indicated, partial credit can be awarded for work shown on various problems. An incorrect answer with no supporting work will receive little to no credit. Make it clear what your final answer to each question is.

Note that there are questions printed on both sides of each page!

| Problem | Total possible | Score |
|---------|----------------|-------|
| 1       | 24             |       |
| 2       | 5              |       |
| 3       | 15             |       |
| 4       | 16             |       |
| 5       | 20             |       |
| TOTAL   | 80             |       |

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1. True/False (24 points, 2 each). No partial credit.

———— (a) The Majority Criterion says "A candidate who gets more than 50% of the votes should be elected."

———— (b) The Condorcet Criterion says "A candidate must win every head-to-head comparison with other candidates in order to win."

———— (c) The Monotonicity Criterion says "If candidate X is the winner of an election and, in a reelection, the only changes in the ballots are changes that favor X, then X should remain the winner of the election."

———— (d) If we are having a vote between only two candidates, there is really only one way to decide who is the winner.

———— (e) In a weighted voting system, someone with more votes automatically has more power.

———— (f) The quota in a weighted voting system needs to be at least 50% of the total votes.

———— (g) A dictator automatically has 100% power according to the Banzhaf power index.

———— (h) Two players with the same number of votes automatically have the same power.

———— (i) When dividing goods among four people, a player has a fair share if they get a piece that they think is worth at least 25% of the total.

———— (j) When we are doing fair division, we assume that no players know anything about what the other players think is fair.

———— (k) The lone-chooser method is used for deciding how to distribute goods that cannot be divided into smaller pieces.

———— (l) In the divider-chooser method, the divider must cut the goods into two equal pieces.

2. Multiple choice (5 points). No partial credit.

———— (a) (2 points) In a weighted voting system, a *dummy* is:

| A | Someone whose vote never affects the outcome | B | A non-critical player in a coalition |
|---|---|---|---|
| C | Someone with zero votes | D | None of the above |

———— (b) (3 points) Which voting system does not need a preference ballot?

| A | The pairwise comparison method | B | The Borda count method |
|---|---|---|---|
| C | The plurality-with-elimination method | D | None of the above |

3. (15 points) The following questions all refer to this preference ballot, which is the outcome of an election between 4 candidates, U, V, W, and X.

| 9 | 10 | 7 | 3 | 6 |
|---|----|---|---|---|
| U | V | W | U | X |
| V | X | X | W | U |
| X | U | U | V | W |
| W | W | V | X | V |

(a) (2 points) Rank these candidates using the extended plurality method.

First ____ Second ____ Third ____ Fourth ____

(b) (3 points) Rank these candidates using the *recursive* plurality method.

First ____ Second ____ Third ____ Fourth ____

(c) (5 points) Rank these candidates using the recursive plurality-with-elimination method.

First ____ Second ____ Third ____ Fourth ____

(d) (2 points) Does this election have a majority candidate? If so, who?

(e) (3 points) Who wins in a head-to-head comparison between $U$ and $X$?

4. (16 points) Weighted voting.
   **The following three questions refer to the weighted voting system [10;5,3,3,3].**

   (a) (3 points) List all the winning coalitions.

   (b) (3 points) How many times is each player critical?

   $P_1$ —— $P_2$ —— $P_3$ —— $P_4$ ——

   (c) (3 points) Find the Banzhaf power index for every player in this voting system. (You can leave your answer as a fraction.)

   $P_1$ —— $P_2$ —— $P_3$ —— $P_4$ ——

   **The following two questions refer to the weighted voting system [4;2,2,1].**

   (d) (4 points) List all the sequential coalitions and identify the pivotal player in each.

   (e) (3 points) Find the Shapley-Shubik power indices of all three players in this voting system. (You can leave your answer as a fraction.)

   $P_1$ —— $P_2$ —— $P_3$ ——

5. (20 points) One more.

   (a) (3 points) We are trying to divide a cake between four people using the lone-divider method. Chooser 1 thinks that piece 1 is worth 10% of the cake, piece 2 is worth 20%, piece 3 is worth 45% and piece 4 is worth 25%. What would this chooser's bid be?

   (b) (3 points) In this same division, Chooser 2 bid on pieces 1 and 2, and Chooser 3 bid on pieces 2 and 4. Describe a fair division of this cake.

   (c) (3 points) We are trying to divide 2 cups of water and 2 cups of sugar. Alfred likes sugar four times as much as much as water. If he was to get 1 cup of sugar, what percentage of the total would he think that was?

   (d) (5 points) We are trying to divide 3 square feet of granite tile and 6 square feet of sandstone tile between three contractors (A,B,C) using the last-diminisher method. Suppose in the first round, contractor A chooses 6 square feet of sandstone tile, and then contractor B has their turn.

   If contactor B likes granite three times as much as sandstone, what do they do?

(e) (2 points) We are using the method of sealed bids to divide up a bell, a book, and a candle among three people (A,B,C). This is the result of their bidding:

|   | Bell | Book | Candle |
|---|------|------|--------|
| A | $9   | $4   | $5     |
| B | $10  | $5   | $3     |
| C | $3   | $3   | $3     |

Who gets which items?

(f) (4 points) How much money does each player lose or gain when we use the method of sealed bids to divide up the items?