Math 1001
Fall 2007
Test 3
12/3/07
Time allowed: 50 minutes

Name: $\qquad$

Student ID: $\qquad$

Section: 10:10 11:15 12:20

This exam includes 5 pages, including this one and a sheet for scratch work. There are a total of 6 questions on the exam, each with multiple parts.

This test is closed book. You are not allowed to consult the text or any notes you have. Scientific calculators are allowed, but not graphing calculators or any other calculators with more functions.
Show your work. Except where specifically indicated, partial credit can be awarded for work shown on various problems. An incorrect answer with no supporting work will receive little to no credit. Make it clear what your final answer to each question is.
Note that there are questions printed on both sides of each page!

| Problem | Total possible | Score |
| :---: | :---: | :---: |
| 1 | 20 |  |
| 2 | 7 |  |
| 3 | 12 |  |
| 4 | 15 |  |
| 5 | 10 |  |
| 6 | 16 |  |
| TOTAL | 80 |  |

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1. True/False ( 20 points, 2 each). No partial credit.
__ (a) A fractal is an object that is self-similar at different levels of magnification.
(b) The Sierpinski gasket has zero area.
(c) Playing the "chaos game" about 5 times in a row gives an interesting picture.
(d) There are shapes in nature that look very similar to fractals.
(e) The Mandelbrot set is formed by starting with a basic shape and applying some construction repeatedly to it.
(f) Multiplying the numbers 10 and 23 together is an example of a random experiment.
(g) The probability of an event is the same as the probability of any outcome which is part of the event.
(h) If the odds of winning are 2 to 1 , that means that you have a $50 \%$ chance of winning.
(i) There are 20 different ways to choose 3 CDs out of a stack of 6 if order doesn't matter.
(j) It is possible to shuffle a deck of cards perfectly 8 times in a row and end up with them back in the order they started in.
2. Multiple choice (7 points). No partial credit.
(a) (2 points) Which of the following shapes can be seen by playing the "chaos game"?

| A | Sierpinski gasket | B | Koch snowflake |
| :--- | :--- | :---: | :--- |
| C | Mandelbrot set | D | Cantor set |

(b) (2 points) You are on a game show with 3 boxes, one with a prize hidden inside and two without, at random. If you guess that the prize is inside box 1 , what is your chance of being correct?

| A | $1 / 3$ | B | $1 / 2$ |
| :--- | :--- | :--- | :--- |
| C | $2 / 3$ | D | None of the above |

(c) (3 points) In the same situation, after you chose the host accidentally reveals that the prize is not in box 2 . Now what is the chance that you picked the correct box?

| A | $1 / 3$ | B | $1 / 2$ |
| :--- | :--- | :--- | :--- |
| C | $2 / 3$ | D | None of the above |

3. (12 points, 3 points each) Add up the following infinite geometric series. State your answer as a fraction, not in decimal form.
(a)

$$
1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots=
$$

$\qquad$
(b)

$$
3+\frac{3}{4}+\frac{3}{16}+\frac{3}{64}+\frac{3}{256}+\cdots=
$$

(c)

$$
3+2+\frac{4}{3}+\frac{8}{9}+\frac{16}{27}+\cdots=
$$

(d)

$$
1+0.01+0.0001+0.000001+\cdots=
$$

$\qquad$
4. (15 points, 3 points each) Calculate the following using arithmetic of complex numbers.
(a)

$$
(3-4 i)+(2+3 i)=
$$

(b)

$$
(1-5 i)-(2+3 i)=
$$

(c)

$$
(1+i) \times(1+i)=
$$

(d)

$$
(1-4 i) \times(2+3 i)=
$$

(e)

$$
(a+b i) \times(a-b i)=
$$

5. (10 points, 2 points each) Independent events.

We roll a die 3 times in a row. For each pair of events listed below, decide whether the two events are independent or not. If they are, mark with a Y; otherwise, mark with an N.

| $\mathbf{Y} / \mathbf{N}$ | Event 1 | Event 2 |
| :--- | :--- | :--- |
|  | First roll is 2 or 3 | Second roll is a 3 |
|  | Rolling three 1s | Second roll is a 5 |
|  | Third roll is a 2 | First two rolls add up to 5 |
|  | All three rolls add up to 4 | First two rolls add up to 10 |
|  | None of the rolls are 6 | None of the rolls are 1 |

6. (16 points) We have an opaque jar filled with 100 marbles - 10 red, 30 blue, and 60 green - that are thoroughly mixed. Our random experiment is: We reach in and pull out two marbles, keeping track of which came out first and which came out second.
(a) (3 points) Write down the sample space for this experiment.
(b) (3 points) What is the probability of getting a red marble followed by a green marble?
(c) (2 points) Express the outcome "neither marble is blue" as a subset of the sample space.
(d) (4 points) What is the probability that neither marble is blue?
(e) (4 points) What is the probability that one of the marbles is green?
