Math 2374
Spring 2008
Exam 3
April 30, 2008
Time Limit: 1 hour

Name (Print): ____________________
Student ID: ____________________
Section Number: ______
Teaching Assistant: ____________________
Signature: ____________________

This exam contains 6 numbered problems on four sheets of paper. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. You are allowed to use one-half of one (doubled-sided) 8.5 inch $\times$ 11 inch sheet of notes.

Show your work, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning.** To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

Note that one or more of these problems may be solved in more than one way.

You may use the major theorems of the course, such as Green's Theorem and Stokes' Theorem, in your reasoning, but you must state that you are using them. (You do not have to state or prove the theorem, though.) Do not use Gauss' Theorem on this test.

1	20 pts	
2	25 pts	
3	20 pts	
4	30 pts	
5	25 pts	
6	20 pts	
TOTAL	140 pts	

1. (20 points) Find

$$\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S},$$

where $\mathbf{F} = (\sin(x^2yz), \sin(xy^2z), \sin(xyz^2))$, and S is the triangle in $\mathbf{R}^3$ with vertices $(0, 0, 5)$, $(0, 4, 0)$, and $(3, 0, 0)$, oriented with upward-pointing normal.

2. (25 points) Let $f(x, y, z) = xy$ and let S be the part of the graph of $z = 4 - x^2$ that lies above the square $0 \leq x \leq 2$, $0 \leq y \leq 2$.

   (a) (15) Find
$$\iint_S f\, dS,$$
   where S is parametrized with upward-pointing normal.

   (b) (10) Find
$$\iint_S f\, dS$$
   as above, except where S has downward-pointing normal.

3. (20 points) Find

$$\iint_D e^{x+y}\, dA,$$

where D is the parallelogram in the xy-plane with vertices $(0,0)$, $(2,1)$, $(-1,-2)$, and $(1,-1)$. (Hint: Use a change of variables that transforms a square in the uv-plane to the parallelogram.)

4. (30 points) (a) (15) Write a parametrization $\Phi(\theta, \phi)$ in standard spherical coordinates for the ellipsoid $9x^2 + 4y^2 + z^2 = 36$. Include the correct bounds for θ and ϕ.

(b) (15) Using any method you like, find an equation for the tangent plane to the ellipsoid at the point where θ and ϕ are both $\frac{\pi}{4}$.

5. (25 points) Consider the sphere $x^2 + y^2 + z^2 = 1$ and the cone $4z^2 = x^2 + y^2$ in the part of $\mathbf{R}^3$ where $z \geq 0$.

(a) (10) Write a parametrization for the curve of intersection between the sphere and the cone.

(b) (15) Suppose that the solid region trapped between the sphere and the cone has density given by $\delta(x, y, z) = z$, in kg/m^3. Find the mass of the solid region.

6. (20 points) Let $\mathbf{F} = 2\mathbf{k}$, and let S be the part of the sphere of radius 1 about the origin in $\mathbf{R}^3$ that lies above the xy-plane. Suppose that S is oriented with upward-pointing normal. Find

$$\iint_S \mathbf{F} \cdot d\mathbf{S}.$$