Math 2374
Spring 2009
Midterm 3
April 29, 2009
Time Limit: 1 hour

Name (Print): ______________________
Student ID: ______________________
Section Number: ______
Teaching Assistant: ______________________
Signature: ______________________

This exam contains 8 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. You are allowed to take one-half of one (doubled-sided) 8.5 inch $\times$ 11 inch sheet of notes into the exam.

Do not give numerical approximations to quantities such as $\sin 5$, π, or $\sqrt{2}$. However, you should simplify $\cos\frac{\pi}{4} = \sqrt{2}/2$, $e^0 = 1$, and so on.

The following rules apply:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning, including the evaluation of definite and indefinite integrals**. To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- **Mysterious or unsupported answers will not receive full credit**. Your work should be mathematically correct and carefully and legibly written.
- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit**; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.
- Full credit will be given only for work that is presented neatly and logically; work scattered all over the page without a clear ordering will receive from little to no credit.

1	25 pts	
2	30 pts	
3	30 pts	
4	30 pts	
5	25 pts	
TOTAL	140 pts	

1. (25 points) Construct a change of variables $T : D^* \to D$ that maps the unit square $D^* = [0,1] \times [0,1]$ to the parallelogram D with vertices $(1,1)$, $(3,3)$, $(2,8)$ and $(0,6)$. Use T to compute the area of D.

2. (30 points) Compute the volume of the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies outside the surface $x^2 + z^2 = 3$.

3. (30 points) Let S be the part of the plane $x+y+z=1$ that lies in the first octant ($x \geq 0$, $y \geq 0$ and $z \geq 0$), with the normal oriented 'upwards' (the $z-$component is positive). Let

$$\mathbf{F}(x,y,z) = (y,\ xyz\cos(z+y+z)e^{2x+3y},\ 0).$$

Using Stokes' Theorem, compute

$$\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}.$$

4. (30 points) The surfaces $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 - z^2 = \frac{1}{2}$ intersect in two circles. Let S be the part of the surface of the sphere that lies between these two circles.

   (a) Parametrize this surface using spherical coordinates. Include the bounds for the parameters in your answer.

(b) For the vector field $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$, and considering the outwards orientation for the normal vector, compute

$$\iint_S \mathbf{r} \cdot d\mathbf{S}.$$

5. (25 points) Show that the vector field $\mathbf{F}(x, y, z) = (2xyz, x^2z - z\sin y, x^2y + \cos y + z)$ is conservative. Find f such that $\mathbf{F} = \nabla f$.