

Math 2374
Fall 2010
Final Exam
December 16, 2010
Time Limit: 3 hours

Name (Print): _____
Student ID: _____
Section Number: _____
Teaching Assistant: _____
Signature: _____

This exam contains 14 pages (including this cover page) and 12 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. You are allowed to take one (doubled-sided) 8.5 inch \times 11 inch sheet of notes into the exam.

Do not give numerical approximations to quantities such as $\sin 5$, π , or $\sqrt{2}$. However, you should simplify $\cos \frac{\pi}{4} = \sqrt{2}/2$, $e^0 = 1$, and so on.

The following rules apply:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning, including the evaluation of definite and indefinite integrals.** To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- **Mysterious or unsupported answers will not receive full credit.** Your work should be mathematically correct and carefully and legibly written.
- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit;** an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.
- Full credit will be given only for work that is presented neatly and logically; work scattered all over the page without a clear ordering will receive from little to no credit.

1	20 pts	
2	25 pts	
3	25 pts	
4	20 pts	
5	25 pts	
6	30 pts	
7	25 pts	
8	20 pts	
9	25 pts	
10	30 pts	
11	25 pts	
12	30 pts	
TOTAL	300 pts	

TA sections:

Section	TA	Discussion time
011	Chen	T 9:05am
012	Chen	T 11:15am
013	Klein	T 1:25pm
014	Klein	T 3:35pm
015	Bu	T 4:40pm
016	Bu	T 6:45pm
021	Bashkirov	Th 8:00am
022	Bashkirov	Th 10:10am
023	He	Th 12:20pm
024	He	Th 2:30pm
025	Lee	Th 4:40pm
026	Lee	Th 6:45pm

1. (20 points) Consider the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$, $f(x, y) = (xe^y, ye^x, e^{x+y^2})$. Suppose $g : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ is a function whose matrix of partial derivatives at $(0, 0, 1)$ is given by

$$(\mathbf{D}g)(0, 0, 1) = \begin{bmatrix} 0 & 1 & -3 \\ 1 & 0 & 5 \end{bmatrix}.$$

Find the matrix of partial derivatives of the function $g \circ f$ at $(0, 0, 0)$.

2. (25 points) Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be the function defined by $f(x, y) = 5xy - x^3 - y^3$.

a) Write a unit vector \mathbf{u} that indicates the direction in which f is decreasing the fastest at the point $(-2, 1)$.

b) Find the directional derivative of f at the point $(-2, 1)$ in the direction \mathbf{u} that you found in part (a).

3. (25 points) Consider the wire parametrized by $\mathbf{c}(t) = (t \cos t, t \sin t, t)$, $1 \leq t \leq 2$.

a) Set up, but **do not evaluate**, the integral for the length of the wire.

b) Suppose the density of the wire at the point (x, y, z) is $\delta(x, y, z) = z$. Find the mass of the wire.

4. (20 points) Compute

$$\int_0^8 \int_{\sqrt[3]{x}}^2 \cos(y^4) dy dx.$$

5. (25 points) Find the volume of the region bounded by the cylinder $x^2 + y^2 = 9$ and the planes $x + y + z = 1$ and $x + y + z = 4$.

6. (30 points) Is $F = (y^2 + 2xz, 2xy, x^2 + 2z)$ the gradient of a function $f(x, y, z)$? If so, find such a function $f(x, y, z)$, and if not, show why not.

7. (25 points) Evaluate

$$\iint \frac{x}{\sqrt{x^2 + y^2}} dx dy$$

over the region, between angles 0 and $\frac{5\pi}{4}$, enclosed by the circles of radius 2 and 3 around the origin.

8. (20 points) Let $F(x, y, z)$ be the vector field

$$(\sin(x)e^y - 3xz^2, \cos(y) - \cos(x)e^y, z^3 + z \sin(y)).$$

Calculate the total flow of this vector field out of cylinder bounded by the surfaces $x = 0$, $x = 9$, and $y^2 + z^2 = 4$.

9. (25 points) Calculate the total work done by the vector field

$$F(x, y) = (xe^x + yx, e^y + x)$$

on an object that travels *clockwise* around the boundary of the half-circle $x^2 + y^2 \leq 4$, $x \geq 0$.

10. (30 points) Suppose we have a fluid with flow given by the vector field

$$F(x, y, z) = (e^z + \sin(x), 2 \sin(y/2), e^{x+y^2})$$

through the rectangle with vertices $(0, 0, 0)$, $(0, 0, 3)$, $(1, 2, 3)$, and $(1, 2, 0)$. Give the total flow through this surface, in the form of both the magnitude of the total flow and a vector pointing in the same direction as the total flow.

11. (25 points) Find the maximum and minimum values of the function $f(x, y) = x(x^2 + y^2 - 1)$ on the disk $x^2 + y^2 \leq 1$.

12. (30 points) Find the maximum and minimum values of the function $f(x, y, z) = x + y - z$ on the ellipsoid with equation $x^2 + 4y^2 + 9z^2 = 49$.