Math 2374	**Name (Print):** ______________
Spring 2007	**Student ID:** ______________
Midterm 1	**Section Number:** ______
February 21, 2007	**Teaching Assistant:** ______________
Time Limit: 1 hour	**Signature:** ______________

This exam contains 8 pages (including this cover page) and 8 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. You are allowed to take one-half of one (doubled-sided) 8.5 inch $\times$ 11 inch sheet of notes into the exam.

Do not give numerical approximations to quantities such as $\sin 5$, π, or $\sqrt{2}$. However, you should simplify $\cos \frac{\pi}{4} = \sqrt{2}/2$, $e^0 = 1$, and so on.

The following rules apply:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning, including the evaluation of definite and indefinite integrals**. To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- **Mysterious or unsupported answers will not receive full credit**. Your work should be mathematically correct and carefully and legibly written.
- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit**; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.
- Full credit will be given only for work that is presented neatly and logically; work scattered all over the page without a clear ordering will receive from little to no credit.

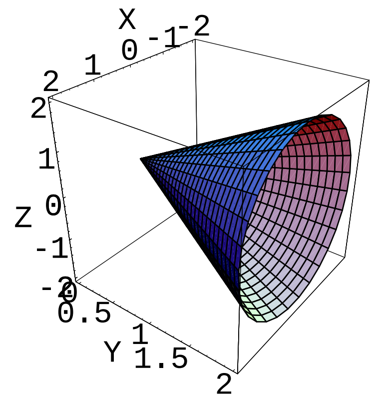
1	25 pts	
2	20 pts	
3	12 pts	
4	20 pts	
5	20 pts	
6	11 pts	
7	12 pts	
8	20 pts	
TOTAL	140 pts	

1. (25 points) Let $g(x, y) = e^{x+y}$ and $\mathbf{r} : \mathbf{R} \to \mathbf{R}^2$ be a function where $\mathbf{r}(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\mathbf{r}'(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Find $F'(0)$ where $F(t) = g(\mathbf{r}(t))$.

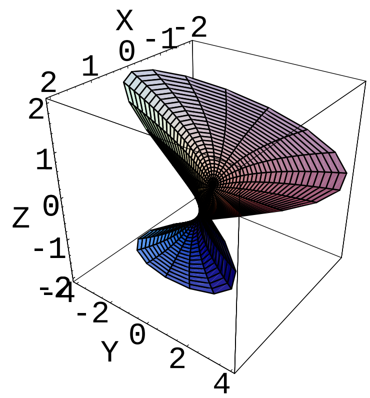
2. (20 points) Find an equation for the plane containing the line parametrized by $(x, y, z) = \mathbf{c}_1(t) = (2 + t, 2t, 1 - t)$ and the line parametrized by $(x, y, z) = \mathbf{c}_2(t) = (3 - t, -2t - 1, t - 6)$. Write your answer in the form $Ax + By + Cz + D = 0$.

3. (12 points) Match the equation with its graph. Give reasons for your choice.

(a) $y = \sqrt{x^2 + z^2}$ (b) $y = x^2 + z^2$ (c) $1 = x^2 + z^2$ (d) $y = x^2 - z^2$



I.



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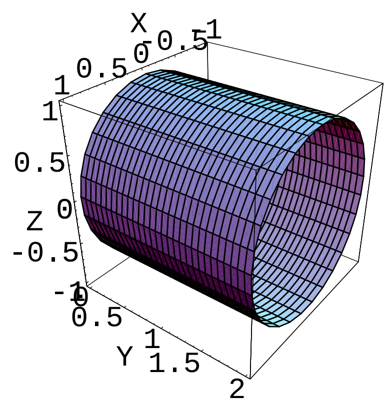

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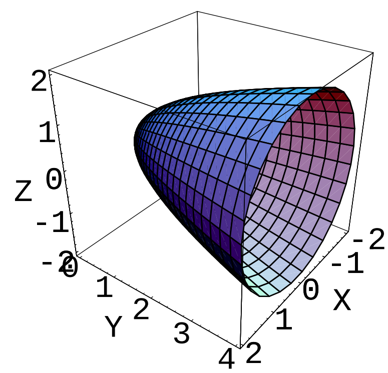


IV.

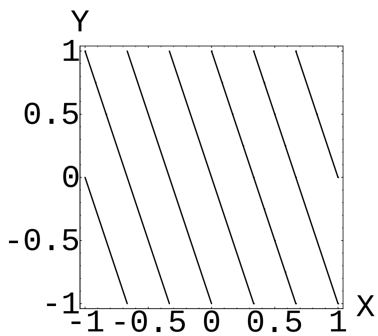
4. (20 points) On a hill which is shaped like the surface $z = \sin(xy)$, a drop of rain falls and lands at the point $(1, 0, 0)$. In which direction along the xy plane would that drop of rain head from that point to get down the hill as quickly as possible? Justify your answer.

5. (20 points) Consider the surface defined by $z = f(x, y) = 2x^2 + xy + y^2 - 3$. Find the equation for the tangent plane at the point $(x, y, z) = (1, 2, 5)$.

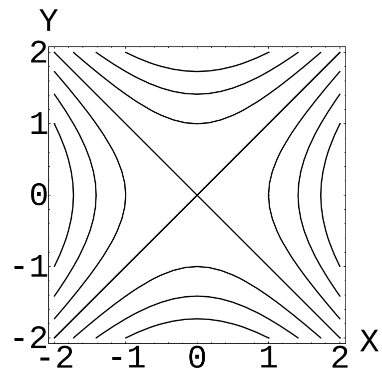
6. (11 points) Consider the function of question 5: $f(x, y) = 2x^2 + xy + y^2 - 3$. Find a linear approximation to $f(x, y)$ at the point $(x, y) = (1, 2)$. Use this linear approximation to estimate $f(1.1, 1.9)$.

7. (12 points) Match the equation with its level curve plot. In each plot, the level curves $c = 0, 1, 2, 3, 4, 5, 6$ are shown. Give reasons for your choice.

(a) $f(x, y) = x^2 + y^2$ (b) $f(x, y) = x^2 - y^2 + 3$
(c) $f(x, y) = \sqrt{x^2 + y^2}$ (d) $f(x, y) = 3x + y + 3$



I.



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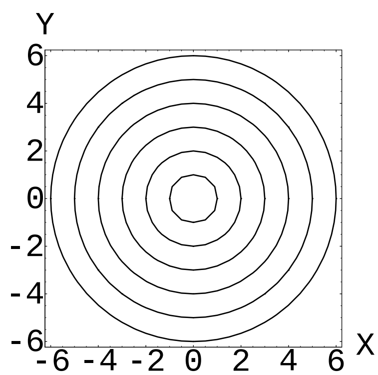

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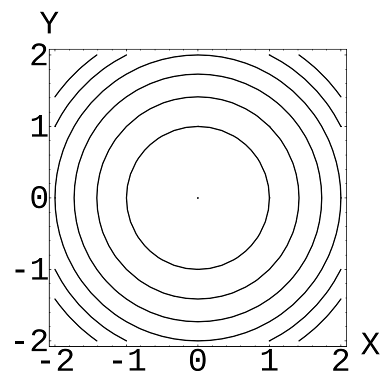

IV.

8. (20 points) For the $g(x, y, z) = z - x^2 + y^2$, the level surfaces will be hyperbolic paraboloids. Find the tangent plane to the level surface $g(x, y, z) = 42$ at the point $(2, 7, -3)$.