Math 2374
Spring 2007
Midterm 3
April 25, 2007
Time Limit: 1 hour

Name (Print): ____________________
Student ID: ____________________
Section Number: ______
Teaching Assistant: ____________________
Signature: ____________________

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. You are allowed to take one-half of one (doubled-sided) 8.5 inch $\times$ 11 inch sheet of notes into the exam.

Do not give numerical approximations to quantities such as $\sin 5$, π, or $\sqrt{2}$. However, you should simplify $\cos \frac{\pi}{4} = \sqrt{2}/2$, $e^0 = 1$, and so on.

The following rules apply:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning, including the evaluation of definite and indefinite integrals**. To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- **Mysterious or unsupported answers will not receive full credit**. Your work should be mathematically correct and carefully and legibly written.
- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit**; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.
- Full credit will be given only for work that is presented neatly and logically; work scattered all over the page without a clear ordering will receive from little to no credit.

1	30 pts	
2	20 pts	
3	20 pts	
4	20 pts	
5	20 pts	
6	30 pts	
TOTAL	140 pts	

1. (30 points) Consider the surface parametrized by $(x, y, z) = \Phi(x, y) = (x, y, 4 - (x^2 + y^2))$ between the planes $z = 1$ and $z = 3$.

   (i) (15 points) Set up the integral to find the surface area.

   (ii) (10 points) In the resulting double integral, change variables to polar coordinates.

   (iii) (5 points) This integral should be easy to evaluate. Do it.

2. (20 points) Compute the integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = (y - z, x - z, x - y)$ and S is the planar surface parametrized by $\mathbf{\Phi}(u, v) = (u - v, u + v, u)$ for $0 \leq u \leq 1$ and $0 \leq v \leq 1$. Orient the surface so the first component of the normal vector is positive.

3. (20 points) Let $\mathbf{F}(x, y) = (y \cos x + 2xe^y, \sin x + x^2 e^y + 5)$.

   (i) (5 points) Verify that $\mathbf{F}$ is a conservative vector field.

   (ii) (15 points) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{s}$ where C is any curve from (0,1) to $(\pi/2, 2)$.

4. (20 points) Consider the integral $\iint_D (4x^2 + 9y^2)dA$ where D is the region bounded by the curve $4x^2 + 9y^2 = 36$.

   (i) (10 points) Let $\mathbf{T}$ be the transformation from a region D^* to D defined by $(x, y) = \mathbf{T}(u, v) = (u/2, v/3)$. Draw both the regions D and D^*.

   (ii) (10 points) Change variables to an integral in u and v. (You need not evaluate the final integral.)

5. (20 points) Let C be the circle parameterized by $(x, y, z) = (\cos t, \sin t, 4)$ for $0 \leq t \leq 2\pi$. Use Stokes' Theorem to calculate the circulation of the vector field

$$\mathbf{F}(x, y, z) = (x + y)\mathbf{i} - (x + y + 2z)\mathbf{j} + (5x - 8z)\mathbf{k}$$

around C, which is the integral $\int_C \mathbf{F} \cdot d\mathbf{s}$. Sketch the curve, your chosen surface, along with a normal vector to show the surface orientation.

6. (30 points) Consider the following triple integral

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{0}^{2-\sqrt{x^2+y^2}} 1\,dz\,dy\,dx$$

(i) (5 points) Describe the solid for which we would be finding its volume with this integral.

(ii) (10 points) Change variables in the integral to cylindrical coordinates.

(iii) (15 points) Change variables in the original integral to spherical coordinates.