

Math 2374  
Fall 2010  
Midterm 3  
December 1, 2010  
Time Limit: 1 hour

Name (Print): \_\_\_\_\_  
Student ID: \_\_\_\_\_  
Section Number: \_\_\_\_\_  
Teaching Assistant: \_\_\_\_\_  
Signature: \_\_\_\_\_

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This exam contains 8 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. You are allowed to take one-half of one (doubled-sided) 8.5 inch  $\times$  11 inch sheet of notes into the exam.

Do not give numerical approximations to quantities such as  $\sin 5$ ,  $\pi$ , or  $\sqrt{2}$ . However, you should simplify  $\cos \frac{\pi}{4} = \sqrt{2}/2$ ,  $e^0 = 1$ , and so on.

The following rules apply:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning, including the evaluation of definite and indefinite integrals.** To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- **Mysterious or unsupported answers will not receive full credit.** Your work should be mathematically correct and carefully and legibly written.
- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit;** an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.
- Full credit will be given only for work that is presented neatly and logically; work scattered all over the page without a clear ordering will receive from little to no credit.

1	20 pts	
2	25 pts	
3	20 pts	
4	25 pts	
5	25 pts	
6	25 pts	
TOTAL	140 pts	

TA sections:

Section	TA	Discussion time
011	Chen	T 9:05am
012	Chen	T 11:15am
013	Klein	T 1:25pm
014	Klein	T 3:35pm
015	Bu	T 4:40pm
016	Bu	T 6:45pm
021	Bashkirov	Th 8:00am
022	Bashkirov	Th 10:10am
023	He	Th 12:20pm
024	He	Th 2:30pm
025	Lee	Th 4:40pm
026	Lee	Th 6:45pm

1. (20 points) Use spherical coordinates to give a parametrization of the ellipsoid determined by the equation  $x^2 + 4y^2 + 9z^2 = 1$ .

2. (25 points) Suppose we have a fluid in  $\mathbb{R}^3$  with flow given by the vector field  $F(x, y, z) = (x, y, 1)$ . Suppose we have a surface parametrized by  $\phi(u, v) = (u + v, v, uv)$  for  $0 \leq u \leq 1$ ,  $2 \leq v \leq 3$ . Find the total rate of flow of fluid through this surface (in the direction given by the normal vector of this parametrization).

3. (20 points) Evaluate the integral

$$\iiint_D e^{x^2+y^2} dx dy dz$$

over the region described by the equations  $x^2 + y^2 \leq 2$ ,  $1 \leq z \leq 5$ .

4. (25 points) Let  $D$  be the region in  $\mathbb{R}^2$  bounded by the curves  $x + y = 2$ ,  $x + y = 4$ ,  $x - y = 0$ ,  $x - y = 4$ . Use the change of coordinates  $x = u + v$ ,  $y = u - v$  to evaluate the integral

$$\iint_D \frac{x - y}{x + y} dx dy.$$

5. (25 points) Given a surface  $S$  described by the equation  $z = g(x, y)$  as  $(x, y)$  varies over a region  $D \subset \mathbb{R}^2$ , **give a parametrization of  $S$  and use it to derive the formula**

$$\text{area}(S) = \iint_D \sqrt{1 + (\partial g/\partial x)^2 + (\partial g/\partial y)^2} \, dx \, dy$$

6. (25 points) Let  $C$  be the perimeter of the triangle in  $\mathbb{R}^3$  with vertices  $(1, 0, 3)$ ,  $(2, 0, 3)$ , and  $(1, 1, 3)$  in order. If  $F(x, y, z) = (xe^x, y^3 + x^2, xz)$ , use Stokes' theorem to evaluate the line integral

$$\int_C F \cdot ds.$$