Math 2374
Spring 2010
Midterm 3
April 22, 2010
Time Limit: 50 minutes
$\qquad$
Name (Print):
Student ID:
Section Number:
Signature:

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. You are allowed to take one-half of one (doubled-sided) 8.5 inch $\times 11$ inch sheet of notes into the exam.

Do not give numerical approximations to quantities $\operatorname{such}$ as $\sin 5, \pi$, or $\sqrt{2}$. However, you should simplify $\cos \frac{\pi}{4}=\sqrt{2} / 2, e^{0}=1$, and so on.

The following rules apply:

- Show your work, in a reasonably neat and coherent way, in the space provided. All answers must be justified by valid mathematical reasoning, including the evaluation of definite and indefinite integrals. To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- Mysterious or unsupported answers will not receive full credit. Your work should be mathematically correct and carefully and legibly written.
- A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.
- Full credit will be given only for work that is presented neatly and logically; work scattered all over the page without a clear ordering will receive from little to no credit.

| 1 | 25 pts |  |
| :---: | :---: | :--- |
| 2 | 25 pts |  |
| 3 | 20 pts |  |
| 4 | 25 pts |  |
| 5 | 20 pts |  |
| 6 | 25 pts |  |
| TOTAL | 140 pts |  |

1. (25 points) Use the spherical coordinates to parametrize the surface

$$
3\left(x^{2}+y^{2}\right)+2 z^{2}=2
$$

with $z \geq x^{2}+y^{2}$.
2. (25 points) Is there a function $f(x, y, z)$ so that $\nabla f(x, y, z)=6 x y(\cos z) \vec{i}+3 x^{2}(\cos z) \vec{j}-$ $3 x^{2} y(\sin z) \vec{k}$ ? If so, find function $f(x, y, z)$.
3. (20 points) Let $\Phi(u, v)=(u-v, u+v, u v)$ and let $D$ be the unit disk in the $u v$ plane. Find the tangent plane of $\Phi(D)$ at $\Phi(1,1)$. Also, find the area of $\Phi(D)$.
4. (25 points) Evaluate

$$
\iint_{S} \vec{F} \cdot d \vec{S}
$$

where $\vec{F}=(x, y,-y)$ and $S$ is the cylindrical surface defined by $x^{2}+y^{2}=1,0 \leq z \leq 1$, with normal pointing out of the cylinder.
5. (20 points) Let $B$ be the region in the first quadrant bounded by the curves $x y=1, x y=3$, $x^{2}-y^{2}=1$, and $x^{2}-y^{2}=4$. Evaluate

$$
\iint_{B}\left(x^{2}+y^{2}\right) d x d y
$$

using the change of variables $u=x^{2}-y^{2}, v=x y$.
6. (25 points) Evaluate

$$
\int_{C}(x+y) d x+(2 x-z) d y+(y+z) d z
$$

where $C$ is the perimeter of the triangle connecting $(2,0,0),(0,3,0)$, and $(0,0,6)$ in that order.

