Sample Exam

Exam rules:
Do not give numerical approximations to quantities such as $\sin 5$, π, or $\sqrt{2}$. However, you should simplify $\cos\frac{\pi}{4} = \sqrt{2}/2$, $e^0 = 1$, and so on.

The following rules apply to all exams:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning, including the evaluation of definite and indefinite integrals**. To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.

- **Mysterious or unsupported answers will not receive full credit**. Your work should be mathematically correct and carefully and legibly written.

- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit**; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

- Full credit will be given only for work that is presented neatly and logically; work scattered all over the page without a clear ordering will receive from little to no credit.

1. Find numbers a, b, and c so that $(1,1,1) = a\mathbf{x} + b\mathbf{y} + c\mathbf{z}$, where $\mathbf{x} = (1,1,0)$, $\mathbf{y} = (1,0,1)$, and $\mathbf{z} = (0,1,1)$.

2. Give a parametrization of a plane through the points $(0,1,2)$, $(1,0,1)$, and $(0,0,0)$.

3. Give a set of two equations in x, y, and z which define the line $(x,y,z) = (1+3t, -2+5t, -7t)$.

4. Find the area of the triangle with vertices $(2, 2)$, $(1, 3)$, and $(-1, 4)$.

5. Find a vector perpendicular both the lines $(1 + t, 2 + t, 3t)$ and $(t, 3, 1 - t)$.

6. Give an equation for the plane containing $(1, 0, 1)$, $(0, 0, 1)$, and $(0, 0, 2)$. Write your answer in the form $Ax + By + Cz + D = 0$.

7. Express the point $(9/4, 12/5)$ in polar coordinates.

8. Does the limit

$$\lim_{(x,y,z)\to(0,0)} \frac{x^2yz}{x^2+y^8+z^2}.$$

exist? If so, calculate it; if not, show why not.

9. Calculate the Jacobian matrix of the function

$$f(x,y,z) = (xye^z + \cos(x^2+y^2), e^{x^2-y^2}, z^4xe^y).$$

10. Suppose that a viscosity at position (x, y, z) is given by the function

$$V(x, y) = x + y^2 + z$$

If you were located at position $(1, 3, -1)$, find the direction that you would need to move in order to *decrease* the viscosity as quickly as possible. Write your answer in the form of a unit vector.

11. Give the range of the function $f(x, y) = (x^2, y)$.

12. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, which of the products BAB^T and B^TAB are defined? Calculate all of those which are defined.

13. Use the chain rule to calculate the Jacobian of $g \circ f$ at the point $(1, 1)$, if $f(x, y) = (x^2, x+xy)$, and $g(u, v) = (u^2 - v^2, u^8 - v, v)$.

14. Find the linear approximation to the $f(x, y) = x^2 + xy + y^2$ near the point $(5, -3)$. Use this linear approximation to estimate $f(5.1, -2.9)$.

15. Calculate the distance between the point $(1, 1, 2)$ and the line $(t, t, 3t)$.