Sample Exam

Exam rules:
Do not give numerical approximations to quantities such as $\sin 5$, π, or $\sqrt{2}$. However, you should simplify $\cos\frac{\pi}{4} = \sqrt{2}/2$, $e^0 = 1$, and so on.

The following rules apply to all exams:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning, including the evaluation of definite and indefinite integrals**. To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.

- **Mysterious or unsupported answers will not receive full credit**. Your work should be mathematically correct and carefully and legibly written.

- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit**; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

- Full credit will be given only for work that is presented neatly and logically; work scattered all over the page without a clear ordering will receive from little to no credit.

1. Suppose you are being carried along the curve $c(t) = (t, 1+t, t^3)$, and at time $t = 3$ you are released and no further forces act on you. Find your position at time $t = 4$.

2. Suppose the curve $c(t)$ has a unit tangent vector $T(t)$ and a speed $v(t)$, so that $c'(t) = v(t)T(t)$. Give a formula for the second derivative $c''(t)$ in terms of v and T.

3. Is the torsion of the curve $c(t) = (t, t^2, t^3)$ positive or negative at $t = 0$?

4. Show that $c(t) = (\cos(t), \sin(t))$ is a flow line of the vector field $F(x, y) = (-y, x)$.

5. Calculate the curl of the vector field $F(x, y, z) = (yz, xz, xy)$.

6. Calculate the divergence of the vector field $F(x, y, z) = (x^2y^2, x^3z^3y, x^2yz^4)$.

7. Find the length of the curve $c(t) = (e^t, \sqrt{2}t, e^{-t})$ between $t = 0$ and $t = 1$.

8. Is the vector field $F(x, y, z) = (xy, yz, -yz - \frac{z^2}{2})$ the curl of another vector field? Why or why not?

9. Find the maximum and minimum of the function $x^2 + 3y^2$ on the circle $x^2 + y^2 \leq 1$.

10. Find the maximum of the function $f(x, y) = \frac{1}{x^4+y^4+1}$.

11. The function $f(x, y, z) = x^2 + xy + y^2 + z^2$ has only one critical point, at $(0, 0, 0)$. Is this point a local maximum, local minimum, or a saddle point?

12. Use the gradient and the Hessian matrix to write a quadratic approximation of the function $f(x, y) = e^x \cos(y)$ at the point $(0, 0)$.

13. Let
$$c(t) = (5\cos(t/5), 3, 5\sin(t/5)).$$
Find the unit tangent T, the unit normal N, the curvature $\kappa(t)$, the binormal B, and the torsion $\tau(t)$.