Math 5285H

Final Exam

No collaboration is allowed. This test is open-book and open-library but no electronic sources may be consulted.

This test is due on or by **Thursday, December 22** by 1pm. Return the exam in my mailbox in the mailroom on the first floor of Vincent Hall, or to my office in Vincent Hall 323 by the same time.

1. Describe one of the Sylow p-subgroups of $GL_2(\mathbb{Z}/p)$.
2. Classify groups of order 85.
3. Suppose V is a vector space over the complex numbers $\mathbb{C}$. Prove that the dimension of V when viewed as a vector space over $\mathbb{R}$ is twice the dimension of V over $\mathbb{C}$.
4. Suppose that G is a simple group of size n, and that H is a proper subgroup of index d. Show that $n \leq d!$. (Hint: Use the action of G on left cosets to construct a homomorphism to a symmetric group, and think about the kernel.)
5. Solve the cubic equation $x^3 = 3x - 1$ as follows. If $x = s + t$, you can collect terms to write $x^3 = 3stx + (s^3 + t^3)$. Equate coefficients in these two cubic polynomials to solve for s^3 and t^3, and then s and t, and then finally x.