Math 5285H

Midterm 2

No collaboration is allowed. This test is open-book and open-library but no electronic sources may be consulted.

This test is due in-class on **Friday, December 9**.

1. Give a matrix expression for the linear operator on $\mathbb{R}^3$ that takes a vector v to its orthogonal projection onto the line generated by the vector $(1, 1, 1)$ of length $\sqrt{3}$. (Recall that the projection of u onto the line generated by v is $\frac{u \cdot v}{||v||^2} v$.) Find the rank of this matrix.

2. Let P_2 be the vector space of polynomials of degree 2 or less with coefficients in $\mathbb{R}$. The set $(x^2, x^2 + x, x^2 + x + 1)$ is a basis of P_2.

   Let $D : P_2 \to P_2$ be the linear operator given by $D(f(x)) = xf'(x)$. Express D as a matrix in terms of the above basis.

3. Give a classification of all the groups of order 39.

4. Suppose that p is a prime that does not divide n and G is a group of order pn. If m is the number of Sylow p-subgroups of G, show that G has exactly $(p - 1)m$ elements of order p.

5. Suppose $p > q > r$ are primes and that G is a group of order pqr.

   (a) Using the previous exercise and the Sylow theorems, show that G has *some* normal Sylow subgroup.

   (b) Continuing the previous problem, suppose that H is a Sylow p-subgroup and that there is a normal Sylow subgroup K of order q or r. Use the Sylow theorems twice, first to show that H is normal in HK and then to conclude that H is normal in G.