

Math 5378, Differential Geometry
Homework 4
Due in-class on **Wednesday, February 20**

Numbered exercises are from Do Carmo, *Differential Geometry of Curves and Surfaces*.

1. Section 2.3, Exercise 2.
2. Section 2.3, Exercise 3.
3. Section 2.3, Exercise 9.
4. Section 2.3, Exercise 11.
5. Section 2.4, Exercise 1.
6. Section 2.4, Exercise 2.
7. Section 2.4, Exercise 4.
8. Consider the following function, defined for all $u, v \in \mathbb{R}^2$.

$$x(u, v) = \left(\frac{2u}{u^2 + v^2 + 1}, \frac{2v}{u^2 + v^2 + 1}, \frac{u^2 + v^2 - 1}{u^2 + v^2 + 1} \right)$$

- (a) Show that the points $(0, 0, 1)$, $(u, v, 0)$, and $x(u, v)$ always lie on a straight line.
- (b) Show that $x : \mathbb{R}^2 \rightarrow S^2$ defines a coordinate chart on the 2-sphere that covers all but the point $(0, 0, 1)$.

(This chart is called *stereographic projection*.)