Math 8301, Manifolds and Topology
Homework 6
Due in-class on **Friday, Oct 19**

1. For an integer n and a real number $R > 0$, find the effect of the map $w \mapsto (Rw)^n : S^1 \to \mathbb{C} \setminus 0$ on π_1.

2. Show that if a polynomial $f(z)$ with complex coefficients has no zeros, then the induced map $\pi_1(S^1, 1) \to \pi_1(\mathbb{C} \setminus \{0\}, f(1))$ sends all elements to the identity.[1]

3. Show that the fundamental group of the n-sphere S^n is trivial for $n > 1$ by directly showing that any loop γ is homotopic to the trivial loop. (Yes, you do have to worry about cases where γ is a space-filling curve.)

4. Suppose you are given a set of objects X and, for any $x, y \in X$, a set $E_{x,y}$. Generalize the definition of a free group by defining the *free groupoid* $\mathcal{F}$ with set X of objects and morphisms which are some type of words in the symbols $E_{x,y} \subset Hom_{\mathcal{F}}(x, y)$ and their inverses. (You may assume, when you need to, that every such word has a unique reduced word associated to it.)

5. A *graph* is a simplicial complex with only vertices and edges, i.e. where no faces have dimension higher than one. A *tree* is a graph, with at least one vertex, such that for any vertices $p \neq q$, there exists a *unique* sequence $e_1, e_2, \cdots, e_n$ of edges such that

    - $e_i \neq e_j$ for $i \neq j$,
    - e_i and e_{i+1} always share a common vertex,
    - p is a vertex of e_1, and
    - q is a vertex of e_n.

    Show that any tree gives rise to a space with trivial fundamental group. (If you want, you can instead show the stronger statement that this space is contractible.)

[1] Combining this with the previous problem and problem 2 on set 4, you can cobble together a proof of the fundamental theorem of algebra. Some textbooks also use the fundamental group to do this, but they don't talk about what happens to non-basepoint-preserving homotopies and so they also have to include something annoying.