Math 8302, Manifolds and Topology II
Homework 3
Due in-class on **Monday, February 25**

1. Let $f : M \to N$ be a map of smooth manifolds, and suppose N is equipped with a covariant vector field (given in local coordinates y^i by $a_i dy^i$). Show that this pulls back to a natural covariant vector field on M.

2. Given a map $f : M \to N$ of smooth manifolds and a vector field on M (given in local coordinates x^i by $a^i \frac{\partial}{\partial x^i}$), explain why this does not push forward to a natural vector field on N. Give an example to illustrate this; if it is a complicated one, you are working too hard.

3. Let M be a smooth manifold. Given a smooth atlas of M, give a smooth atlas on the tangent bundle TM. Show that a smooth function $f : M \to N$ gives rise to a smooth function $df : TM \to TN$.

4. Suppose M is a smooth manifold and $p : Y \to M$ is a covering map. Show that Y can be given the structure of a smooth manifold so that the projection map p is smooth.

5. Suppose that M is a smooth manifold and f is a smooth function from M to $\mathbb{R}^n \setminus \{0\}$. Give necessary and sufficient conditions for the smooth map
$$p \mapsto f(p)/||f(p)||$$
from M to S^{n-1} to be
   (a) a submersion,
   (b) an immersion, or
   (c) a local diffeomorphism.