Math 8306, Algebraic Topology
Homework 1
Due in-class on **Monday, September 15**

1. For any space X and point $p \in X$, there is a constant map $c_p : \Delta^1 \to X$ sending all points to p. Viewing this as a 1-dimensional chain in X, show that it is always a boundary.

2. Given any path $\sigma : \Delta^1 \to X$, write $\overline{\sigma}$ for the same path in the opposite direction (in coordinates, $\overline{\sigma}(t) = \sigma(1-t)$). Show that the chain $\sigma + \overline{\sigma}$ is always a boundary. Explain (briefly) some of the extra complications in trying to do something equivalent for maps $\Delta^2 \to X$.

3. Given two paths $\sigma, \gamma : \Delta^1 \to X$ such that $\sigma(1) = \gamma(0)$, we can form a path composite by

$$\sigma * \gamma(t) = \begin{cases} \sigma(2t) & \text{if } 0 \leq t \leq 1/2 \\ \gamma(2t-1) & \text{if } 1/2 \leq t \leq 1. \end{cases}$$

   Show that the chain $\sigma + \gamma - (\sigma * \gamma)$ is always a boundary.

4. Using the previous exercises, explain why an element in $H_1(X)$ can always be represented by a sum of closed loops $S^1 \to X$ with some integer coefficients. (Warning: the same is not even close to true for $H_n(X)$ when $n > 1$.)

5. Given maps $f : A \to B$ and $g : B \to C$ of abelian groups, show that there is an exact sequence

$$0 \to ker(f) \to ker(gf) \to ker(g) \to coker(f) \to coker(gf) \to coker(g) \to 0.$$