Math 8307, Algebraic Topology II
Homework 2
Due in-class on **Wednesday, February 4**

1. Suppose $X = S^1$ with base point $*$ and $A \subset S^1$ is a subspace (containing $*$) with exactly $k > 0$ points. Compute $\pi_n(X, A, *)$ for all $n \geq 1$.

2. Find an example of a pair of spaces $A \subset X$ with basepoint $*$ so that the map $\pi_1(X, *) \to \pi_1(X, A, *)$ cannot possibly be a group homomorphism.

3. Suppose X is a connected space and let $f : S^n \to X$ be any map. Show that f can be extended to a map $D^{n+1} \to X$ if and only if the image of f in $\pi_n(X, f(*))$ is zero.

4. Suppose f is as in the problem and $g, h : D^{n+1} \to X$ are two extensions of f, i.e. $g|_{S^n} = h|_{S^n} = f$. Construct a "difference" $g - h \in \pi_{n+1}(X, f(*))$, and show that there is a homotopy $H : D^{n+1} \times [0,1] \to X$ from g to h that fixes the boundary S^n if and only if this difference is zero.