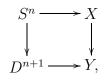
## Math 8307, Algebraic Topology II Homework 4 Due in-class on **Wednesday, February 18**

1. A map  $f: X \to Y$  is called an *acyclic Serre fibration* if, whenever we have a commutative diagram



we can find a lift to a map  $D^{n+1} \to X$  to make the diagram commute. Show that acyclic Serre fibrations are, in particular, Serre fibrations.

- 2. Show that an acyclic Serre fibration gives an isomorphism on homotopy groups.
- 3. Suppose X is a CW-complex whose cells are of dimension d or less and Y is a space with  $\pi_n(Y) = 0$  for  $n \leq d$ . Show that any map  $X \to Y$  is null-homotopic.
- 4. A connected space Y that has only one nonzero homotopy group,

$$\pi_d(Y, y) = \begin{cases} G & \text{if } d = n, \\ 0 & \text{otherwise,} \end{cases}$$

is called an Eilenberg-Maclane space K(G, n). Show that, for any CW-complex X, the set [X, K(G, n)] only depends on the quotient  $X^{(n+1)}/X^{(n-2)}$  of the (n + 1)-skeleton by the (n - 2)-skeleton.