

Math 8307, Algebraic Topology II
Homework 7
Due in-class on **Wednesday, March 25**

1. The special unitary group $SU(n)$ is the subgroup of $U(n)$ consisting of matrices of determinant 1, and there are fibration sequences

$$SU(n) \rightarrow U(n) \rightarrow S^1$$

where the last map is the determinant. Using this, the Serre spectral sequence, and the Hurewicz theorem, compute, $\pi_k U(n)$ for $k \leq 3$.

2. Knowing $SO(3) \cong \mathbb{R}P^3$, compute $H^*(SO(4))$, together with its cup product.
3. Using the Serre spectral sequence and the path-loop fibrations

$$K(A, n) \rightarrow * \rightarrow K(A, n + 1),$$

show that the rational cohomology groups

$$H^*(K(\mathbb{Z}/m, n); \mathbb{Q})$$

are trivial for all $m, n > 0$.

4. Compute the rational cohomology groups

$$H^*(K(\mathbb{Z}, n); \mathbb{Q})$$

for all n .