Euler classes for higher gerbes

Chaque gerbe a sa gerbille
Charles-Louis de Secondat, Baron de La Bréde et de Montesquieu

In the late 1970's Ravenel and Wilson calculated the extraordinary K-theories of Eilenberg-Mac Lane spaces, and showed in particular that

$$K(l)^* H(l+1, \mathbb{Z})$$

is a one-dimensional formal group of multiplicative type (at least, if $p \neq 2$?). Strickland and others (cf eg [8]) re-interpreted their results in terms of exterior powers of the Dieudonné module of $K(l)^* H(\mathbb{Z}, 2)$, and more recently Peterson [6] has extended these results to E-theory.

This seems to define a natural homomorphism

$$\text{gerbe}(l) : H^l(X, \mathbb{C}^\times) \to \text{Gl}_1(K(l)(X) := (1 + \tilde{K}(l)^0(X))^\times \ ,$$

via

$$H(\mathbb{C}^\times, l) \to H(\mathbb{Z}, l+1) \to \text{Gl}_1(K(l))$$

(cf [1]). When $l = 1$ we get the first K-theory (mod p) Chern class of a flat complex line bundle, and when $l = 2$ we get an invariant of Azumaya algebra bundles [2]: a kind of Euler clas or support cycle, conceivably related [3] to physicists' D-branes.

The classical map

$$H^{l+1}(X, \mathbb{C}^\times) \to H^l(LX, \mathbb{C}^\times)$$

(ie from gerbes to line bundles, when $l = 1$) suggests asking how the transformation above behaves on free loopspaces [5]; more generally, an orientable codimension one submanifold (eg $M_0 = \partial M_1 \subset M_1$) defines a Pontrjagin-Thom transfer $H^*(M_0) \to H^{*+1}(M_1)$ and hence a homomorphism $H^*(M_0) \to H^*(LM_1)$, possibley relevant to higher-dimensional topological field theories.

Behind this lies the deeper question:

• **Are these constructions of any use in the theory of higher gerbes**,

as for example in [3 §7.2.2]?

Some referemces

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