18.704 Writing Project

Outline of the writing project

As I mentioned in the course outline, your major project for this course is an expository paper.

The goal of this project is not necessarily for you to do original research. The goal is for you to learn something new, and write a paper to explain what you've learned. Rather than get into the ugly details of proofs, you want to summarize the main mathematical ideas. Your target audience is your group of peers. Try to imagine writing a paper that would have helped *you* understand the topic. (Trust me, this is harder than you think.)

Because the purpose of the project is communication, your goal should be to make your explanation as clear and understandable as possible. Follow the rules of grammar, including in equations. If you read your writing out loud, it should still sound correct. Make your language precise; this makes it easier to understand. Write simply and directly. Write out only the equations you need in order to get the point across, because reading pages of equations is exhausting. Organize your paper to make it easier to follow.

The paper should be roughly 10 pages, and written in some kind of TeX; if you prefer, you can use some other flavor such as LaTeX. Your submissions will be in TeX code to me.

Here is a breakdown of what you need to do.

- Pick a topic and submit your choice to me by April 7th. I'll include a list of possibilities.
- Read and learn something about your topic. Use multiple sources if you can! It often helps you understand if you get several different points of view.
- Submit a draft of your paper to me for a first edit by April 21st.
- I'll do any number of edits that you feel you need, but your final submission is due by May 9th.

Topic choices

If there is something that you are interested in but which is not on this list, I would be happy to hear it. However, all topic choices need to be approved by me before you start.

1. Representations of groups over fields such as $\mathbb{Q}$ or $\mathbb{R}$. As you proved on an assignment question, some properties are special to representations over $\mathbb{C}$. Find out what a semisimple algebra is and describe how Wedderburn's theorem tells you the structure of them.

2. Actions on abelian groups and group cohomology. Instead of having a group act on a vector space, you could have it act on an abelian group. Describe how these actions correspond to modules over $\mathbb{Z}[G]$ and how Maschke's theorem can fail. Describe the group $\mathrm{Ext}(M, N)$ and how it classifies extensions of one G-module by another.

3. Symmetric polynomials and Newton's identities. We described the symmetric and alternating square of a representation, and then gave formulas for their characters. Explain how the formulas for higher symmetric and alternating powers are written in terms of things called elementary symmetric polynomials and power sums. In particular, you could describe how the power sums give tise to the "Adams operations" on representations.

4. Symmetric polynomials and Schur polynomials. Schur polynomials are polynomials that correspond to integer partitions, and they have an explicit form as a determinant. Describe the elementary symmetric polynomials and how the Schur polynomials can be rewritten in terms of symmetric polynomials. Explain how the characters of the symmetric group appear in this rewriting process.

5. A historical essay detailing some of the development of representation theory. For instance, there's a book by Charles Curtis called "Pioneers of representation theory" that talks about the early development of the subject around the beginning of the 20th century; consider choosing a topic to exposit from there.

6. Representation theory of Lie algebras. Define a Lie algebra and what a representation of one is. Examine in detail the 3-dimensional Lie algebra $\mathfrak{sl}_2(\mathbb{R})$ and the classification of its representations.

The next few topics are more tentative, since they require more background.

7. Applications of representation theory to chemistry or physics.

8. The Noether normal basis theorem (requiring some background in field theory). The Noether normal basis theorem gives a description of a finite field extension as a representation of the Galois group. Give an exposition of the proof of this theorem.

9. Representations of other kinds of groups. Most of the results about representations of finite groups are also true for some other groups, such as compact groups. Describe how the proofs of the main results need to be modified to fit this new setup, and what form character theory takes.