

18.704 Problem Set 1

Due Friday, Feb. 17, at 3pm in 2-171

1. The *quaternion group* Q_8 has the eight elements

$$1, -1, i, -i, j, -j, k, -k,$$

and multiplication is given by the rules

$$i^2 = j^2 = k^2 = -1,$$

$$ij = -ji = k,$$

$$jk = -kj = i,$$

$$ki = -ik = j.$$

Find all the subgroups of Q_8 , and show that the commutator subgroup $[Q_8, Q_8]$ is $\{\pm 1\}$.

2. Find all homomorphisms $Q_8 \rightarrow \mathbb{C}^\times$.
3. Suppose G is a finite group with subgroups H and K . A *double coset* is a subset of G of the form

$$\{h x k \mid h \in H, k \in K\}$$

for some $x \in G$. We denote this double coset by HxK . Does the order of a double coset always divide the size of the group $|G|$? Either prove this is true, or find a counterexample.