18.704 Problem Set 5

Due Friday, May 12, at **3pm** in 2-171

At least one of your answers must be typeset in TEX.

(You don't need to submit the TEX code.)

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1. Suppose $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_r)$ is a partition, and $\mu = \lambda^t$ is its conjugate partition. Show that the only way to fill in the table

	μ_1	μ_2	μ_3	$\ldots$	μ_r
λ_1					
λ_2					
λ_3					
$\vdots$					
λ_s					

with zeros and ones, so that the row sums equal the λ_i and the column sums equal the μ_i, is to "fill in the Young diagram." (Hint: Try induction on the number of rows.)

2. Suppose λ and λ' are any two partitions. Recall that $\lambda \leq \lambda'$ if and only if the following equations are true.

$$\begin{aligned} \lambda_1 &\leq \lambda'_1 \\ \lambda_1 + \lambda_2 &\leq \lambda'_1 + \lambda'_2 \\ \lambda_1 + \lambda_2 + \lambda_3 &\leq \lambda'_1 + \lambda'_2 + \lambda'_3 \\ &\vdots \end{aligned}$$

Sketch a proof that $\lambda \leq \lambda'$ if and only if we can obtain the Young diagram for λ by taking the Young diagram for λ' and moving boxes downwards to lower rows.

3. Suppose that we have partitions λ and μ, and that there is a way to fill in the table

	μ_1	μ_2	μ_3	$\ldots$	μ_r
λ_1					
λ_2					
λ_3					
$\vdots$					
λ_s					

with zeros and ones so that the row sums are the λ_i and the column sums are the μ_i. Using the result from the last question, show that $\lambda \leq \mu^t$. (Hint: What happens if you push all of the 1s in the table to the left? To the top?)