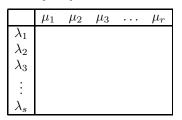
## 18.704 Problem Set 5

Due Friday, May 12, at **3pm** in 2-171

## At least one of your answers must be typeset in T<sub>E</sub>X.

(You don't need to submit the T<sub>E</sub>X code.)

1. Suppose  $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_r)$  is a partition, and  $\mu = \lambda^t$  is its conjugate partition. Show that the only way to fill in the table



with zeros and ones, so that the row sums equal the  $\lambda_i$  and the column sums equal the  $\mu_i$ , is to "fill in the Young diagram." (Hint: Try induction on the number of rows.)

2. Suppose  $\lambda$  and  $\lambda'$  are any two partitions. Recall that  $\lambda \leq \lambda'$  if and only if the following equations are true.

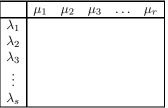
$$\lambda_1 \leq \lambda'_1$$
  

$$\lambda_1 + \lambda_2 \leq \lambda'_1 + \lambda'_2$$
  

$$\lambda_1 + \lambda_2 + \lambda_3 \leq \lambda'_1 + \lambda'_2 + \lambda'_3$$
  
.

Sketch a proof that  $\lambda \leq \lambda'$  if and only if we can obtain the Young diagram for  $\lambda$  by taking the Young diagram for  $\lambda'$  and moving boxes downwards to lower rows.

3. Suppose that we have partitions  $\lambda$  and  $\mu$ , and that there is a way to fill in the table



with zeros and ones so that the row sums are the  $\lambda_i$  and the column sums are the  $\mu_i$ . Using the result from the last question, show that  $\lambda \leq \mu^t$ . (Hint: What happens if you push all of the 1s in the table to the left? To the top?)