In respectful memory of Professor Michael Marinov

SUPERFIELD ALGORITHMS FOR TOPOLOGICAL FIELD THEORIES

IGOR BATALIN[†] and ROBERT MARNELIUS
Department of Theoretical Physics,
Chalmers University of Technology, Göteborg University,
S-412 96 Göteborg, Sweden

A superfield algorithm for master actions of a class of gauge field theories including topological ones in arbitrary dimensions is presented generalizing a previous treatment in two dimensions. General forms for master actions in superspace are given, and possible theories are determined by means of a ghost number prescription and the master equations. The resulting master actions determine the original actions together with their gauge invariances. Generalized Poisson sigma models in arbitrary dimensions are constructed by means of this algorithm, and other simple applications in low dimensions are given including a derivation of the non-abelian Chern-Simon model.

Contents

[†] On leave of absence from P.N. Lebedev Physical Institute, 117924 Moscow, Russia

1 The basic formulation

In our paper[1] we gave a superfield algorithm for a class of master actions in two dimensions by means of which we derived generalized Poisson sigma models. (A superfield form of the master action for the ordinary Poisson sigma models[2,3] was given by Cattaneo and Felder.[4]) In this paper we generalize this algorithm to arbitrary dimensions. This provides then for a general framework for topological field theories and generalizations. All these models share the following properties: i) The equations of motion are of first order in the derivatives. ii) They are directly defined in terms of their corresponding master actions. iii) There are general simple rules for how the original actions are obtained from these master actions. iv) The quantum theory of the models are obtained by a gauge fixing of the master actions. For other related works see also Refs. 5−11.

The master actions corresponding to an n-dimensional field theory will be entirely expressed in terms of fields on a supermanifold of dimension (n, n), *i.e.* half of the coordinates are bosonic, even ones, and half are fermionic, odd ones. To begin with we consider a class of master actions expressed in terms of pairs of superfields. They are

$$\Sigma[\Phi, \Phi^*] = \int d^n u d^n \tau \mathcal{L}_n(u, \tau)\,, \tag{1}$$

where the Lagrangian densities $\mathcal{L}_n(u, \tau)$ are given by

$$\mathcal{L}_n(u, \tau) \quad = \Phi^*_A(u, \tau) D\Phi^A(u, \tau)(-1)^{\varepsilon_A + n} - S(\Phi(u, \tau), \Phi^*(u, \tau))\,, \tag{2}$$

where u^a, $a = 1, 2, \ldots, n$ are bosonic coordinates on the base space and τ^a, $a = 1, 2, \ldots, n$, the corresponding fermionic ones. D is the odd de Rham differential

$$D \equiv \tau^a \partial_a, \quad \partial_a \equiv \frac{\partial}{\partial u^a} \quad \Rightarrow \quad D^2 = 0\,. \tag{3}$$

Since Σ is required to be even and since the measure $d^n\tau$ is even or odd depending on whether n is even or odd, it follows that the Grassmann parity of the Lagrangian density is n, *i.e.* $\varepsilon(\mathcal{L}_n(u, \tau)) = n$. This implies that $\varepsilon(S) = n$ and $\varepsilon(\Phi^*_A) = \varepsilon_A + 1 + n$, where $\varepsilon_A \equiv \varepsilon(\Phi^A)$. (For even n the form of $\mathcal{L}_n$ in (2) is a generalization of the two dimensional treatment in Ref. 1, and for odd n it is a generalization of the one-dimensional expression given in Refs. 6 and 11. Such superfield forms for master actions for topological field theories were also proposed.[5,6])

The master actions (1) are required to satisfy the classical master equation

$$(\Sigma, \Sigma) = 0\,, \tag{4}$$

where the antibracket is defined by

$$\begin{aligned}(F, G) \equiv \int & F \, \frac{\overleftarrow{\delta}}{\delta \Phi^A(u,\tau)} \, (-1)^{n\varepsilon_A} d^n u d^n \tau \, \frac{\overrightarrow{\delta}}{\delta \Phi^*_A(u,\tau)} \, G \\ & - (F \leftrightarrow G)(-1)^{\big(\varepsilon(F)+1\big)\big(\varepsilon(G)+1\big)}\,,\end{aligned} \tag{5}$$

which in turn can be defined by[12]

$$(F, G) = \Delta(FG)(-1)^{\varepsilon_F} - (\Delta F)G(-1)^{\varepsilon_F} - F(\Delta G) \tag{6}$$

in terms of the basic odd, nilpotent Δ-operator

$$\Delta \equiv \int d^n u d^n \tau \, (-1)^{(n+1)\varepsilon_A} \, \frac{\delta}{\delta \Phi^A(u,\tau)} \, \frac{\delta}{\delta \Phi^*_A(u,\tau)} \;. \tag{7}$$

This Δ-operator also differentiates the antibracket (5) according to the Leibnitz rule, *i.e.*

$$\Delta(F, G) = (\Delta F, G) + (F, \Delta G)(-1)^{\varepsilon_F + 1}. \tag{8}$$

The functional derivatives in (5) and (7) are defined by the properties

$$\begin{aligned}&\frac{\delta}{\delta \Phi^B(u,\tau)} \, \Phi^A(u',\tau') = \delta^A_B \, \delta^n(u-u')\delta^n(\tau-\tau') = \Phi^A(u,\tau) \, \frac{\overleftarrow{\delta}}{\delta \Phi^B(u',\tau')} \;, \\ &\varepsilon\left(\frac{\delta}{\delta \Phi^A}\right) = \varepsilon\left(\frac{\overleftarrow{\delta}}{\delta \Phi^A}\right) = \varepsilon_A + n\,, \\ &F \, \frac{\overleftarrow{\delta}}{\delta \Phi^A(u,\tau)} \, d^n\tau = (-1)^{\varepsilon_A(\varepsilon(F)+1)} d^n \tau \, \frac{\delta}{\delta \Phi^A(u,\tau)} \, F\,,\end{aligned} \tag{9}$$

where the delta-function in the odd coordinates τ^a satisfies

$$\int f(\tau') \, \delta^n(\tau - \tau') \, d^n \tau' = f(\tau) = \int d^n \tau' \, \delta^n(\tau' - \tau) f(\tau')\,. \tag{10}$$

An explicit calculation of the antibracket of the master actions (1) yields

$$\frac{1}{2}(\Sigma,\Sigma) = \int d^n u d^n \tau \left(D\mathcal{L}_n(u,\tau) + \frac{1}{2}(S,S)_n(u,\tau) \right), \tag{11}$$

where we have introduced the local n-bracket

$$(f,g)_n \equiv f \frac{\overleftarrow{\partial}}{\partial \Phi^A} \frac{\overrightarrow{\partial}}{\partial \Phi^*_A} g - (f \leftrightarrow g)(-1)^{(\varepsilon(f)+1+n)(\varepsilon(g)+1+n)}, \tag{12}$$

where f and g are local functions of $\Phi^A(u,\tau)$ and $\Phi^*_A(u,\tau)$. For even n this is an antibracket and for odd n it is a Poisson bracket. In other words Φ^A and Φ^*_A are canonical conjugate field variables on an antisymplectic manifold for even n and on a symplectic manifold for odd n. The condition (4) combined with (11) yields

$$\int d^n u d^n \tau D\mathcal{L}_n(u,\tau) = 0\,, \tag{13}$$

which determines the allowed boundary conditions, and

$$(S,S)_n = 0\,. \tag{14}$$

Furthermore, we have from (1) and (7)

$$\Delta\Sigma = 0\,, \tag{15}$$

since the τ-part yields a factor zero. (As usual we believe that the bosonic part can be regularized appropriately.) Therefore, Σ also satisfies the quantum master equation

$$\frac{1}{2}(\Sigma,\Sigma) = i\hbar\Delta\Sigma\,, \tag{16}$$

when (13) and (14) are satisfied, which means that no quantum corrections of the measure in the path integral are required for these models.

If one treats the master action Σ in (1) as an ordinary action, then the equations of motion are

$$D\Phi^A = (S,\Phi^A)_n\,, \quad D\Phi^*_A = (S,\Phi^*_A)_n\,, \tag{17}$$

the consistency of which again requires (14). Notice that the equation of motion for S then becomes $DS = 0$. Notice also the relations

$$\begin{aligned} \left(\Sigma,\Phi^A\right) &= (-1)^n \left(D\Phi^A - (S,\Phi^A)_n\right), \\ \left(\Sigma,\Phi^*_A\right) &= (-1)^n \left(D\Phi^*_A - (S,\Phi^*_A)_n\right). \end{aligned} \tag{18}$$

The superfields above can be Taylor expanded in the odd τ-coordinates in such a fashion that the coefficients are fields and antifields in the ordinary sense on the base manifold. Or in other words, such that the antibracket (5) and the Δ-operator (7) have their conventional forms:

$$(F,G) = \sum_r \int F \, \frac{\overleftarrow{\delta}}{\delta \Phi^{rA}(u)} \, d^n u \, \frac{\overrightarrow{\delta}}{\delta \Phi^*_{rA}(u)} \, G - (F \leftrightarrow G)(-1)^{\left(\varepsilon(F)+1\right)\left(\varepsilon(G)+1\right)} \,,$$

$$\Delta \equiv \sum_r \int d^n u \, (-1)^{\varepsilon_{rA}} \, \frac{\delta}{\delta \Phi^{rA}(u)} \, \frac{\delta}{\delta \Phi^*_{rA}(u)} \,, \qquad \varepsilon_{rA} \equiv \varepsilon(\Phi^{rA}) \,, \tag{19}$$

where $\Phi^{rA}(u)$ and $\Phi^*_{rA}(u)$ are the coefficient fields (r denotes antisymmetric u-indices). The antibracket in (19) must yield

$$(\Phi^A(u,\tau), \Phi^*_B(u',\tau')) = (-1)^{n\varepsilon_A} \delta^A_B \delta^n(u-u') \delta^n(\tau-\tau') \,, \tag{20}$$

which trivially follows from Eq. (5). An explicit construction is given in Appendix B.

So far we have given superfield actions in arbitrary dimensions which under certain conditions satisfy the master equations. In the following we will always require Σ to actually satisfy the master equation, *i.e.* we require the boundary conditions to be consistent with (13) and the local equation (14) to be satisfied. In order to determine the class of gauge field theories which have such master actions we need also to prescribe ghost numbers to the field. We generalize then the prescription given in Ref. 1 (such ghost numbers were also considered in Refs. 4, 6, 8 and 11). We choose the odd coordinates τ^a to have ghost number plus one, which implies that D in (3) has ghost number plus one. Since we require the master action Σ to have ghost number zero and since the measure $d^n\tau$ has ghost number $-n$ the Lagrangian density $\mathcal{L}_n$ in (2) must have ghost number n, *i.e.*

$$gh(\Sigma) = 0 \,, \quad gh(d^n\tau) = -n \,, \qquad \Rightarrow \qquad (\mathcal{L}_n) = n \,. \tag{21}$$

The form (2) of $\mathcal{L}_n$ leads us to the following general rule for the superfields and the local function S:

$$gh(\Phi^A) + gh(\Phi^*_A) = n-1 \,, \quad gh(S) = n \,. \tag{22}$$

We assume that S is given by a power expansion in the superfields Φ^A and Φ^*_A, in which case this ghost number prescription will restrict the possible terms in S. In such an analysis it is convenient to use the following convention:

$$gh(\Phi^*_A) \geq gh(\Phi^A) \,, \tag{23}$$

which in itself does not impose any restriction. Notice that Φ^A and Φ^*_A

can only have equal ghost number in odd dimensions. (More precisely for $n = 2m+1$ we can have the ghost numbers $gh(\Phi^A) = gh(\Phi^*_A) = m$.) Since the local bracket (12) satisfies

$$\big(\Phi^A(u,\tau), \Phi^*_B(u,\tau)\big)_n = \delta^A_B\,, \tag{24}$$

it follows from the ghost number prescription (22) that the local bracket $(\ ,\ )_n$ in itself carries ghost number $1-n$. From (20) and the fact that $\delta^n(\tau-\tau')$ carries ghost number n, it follows that the antibracket (5) or, equivalently, (19) carries ghost number plus one which is the standard ghost number prescription for the antibracket in the field-antifield formalism.

We have a consistent master action Σ given by (1) if S satisfies $gh(S) = n$ and $(S,S)_n = 0$ and the set of superfields satisfies (22) and the boundary conditions compatible with (13). From such a master action we can then extract the original model according to the following rules. First, remove the integration measure. Then perform the following replacements:

$$\begin{array}{rcl} D & \longrightarrow & \text{exterior derivative } d\,, \\ \Phi^A,\ gh(\Phi^A) = k \geq 0\,, & \longrightarrow & k\text{–form field } \phi^A,\ \varepsilon(\phi^A) = \varepsilon_A + k\,, \\ \Phi^*_A,\ gh(\Phi^*_A) = (n-1-k) \geq 0\,, & \longrightarrow & (n-1-k)\text{–form field } \phi^*_A,\ \varepsilon(\phi^*_A) = \varepsilon_A + k\,, \\ \Phi^A,\ gh(\Phi^A) < 0\,, & \longrightarrow & 0 \\ & \text{or} & \\ \Phi^*_A,\ gh(\Phi^*_A) < 0\,, & \longrightarrow & 0\,, \\ \text{ordinary multiplication} & \longrightarrow & \text{wedge products,} \end{array} \tag{25}$$

where the form fields, ϕ^A and ϕ^*_A, are fields on the n-dimensional u-space. These rules are easily extracted from the fact that the original fields, $\phi^A(u)$ and $\phi^*_A(u)$, are component fields with ghost number zero of the superfields, $\Phi^A(u,\tau)$ and $\Phi^*_A(u,\tau)$. (Signs depend on how the coefficient fields are defined precisely.)

The superfield master actions determine also the gauge invariance of the original action. The appropriate gauge transformations can be obtained by following steps. First identify the components of the superfields that are antifields to the original fields. These antifields have ghost number minus one and if Φ^A contains the field then Φ^*_A contains the corresponding antifield or vice versa. Identify then the terms in the master Lagrangian $\int d^n\tau \mathcal{L}_n$ which are linear in these antifields. The terms in the coefficients of these antifields, which contain no antifields, represent then the gauge transformations of the field. The ghost fields with ghost number plus one are to be interpreted then as gauge parameters.

2 Master actions in terms of general superfields

We have generalized[1] the two-dimensional treatment along the lines of the generalized antisymplectic formulation.[13] Here we give the corresponding treatment in arbitrary dimensions. Let the superfields $Z^I(u,\tau)$, $\varepsilon(Z^I)\equiv\varepsilon_I$, represent arbitrary coordinates on an (anti)symplectic manifold. The local brackets (12) are then defined by

$$(f,g)_n \equiv f(Z(u,\tau))\,\frac{\overleftarrow{\partial}}{\partial Z^I}E^{IK}(Z(u,\tau))\frac{\overrightarrow{\partial}}{\partial Z^K}\,g\,(Z(u,\tau))\,, \tag{26}$$

which implies

$$(Z^I(u,\tau),Z^K(u,\tau))_n = E^{IK}(Z(u,\tau))\,. \tag{27}$$

The functions E^{IK} have the properties

$$\begin{aligned}\varepsilon\left(E^{IK}\right) \;&=\; \varepsilon_I+\varepsilon_K+1+n,\\ E^{KI} = &\;-E^{IK}(-1)^{(\varepsilon_I+1+n)(\varepsilon_K+1+n)}\,.\end{aligned} \tag{28}$$

These properties follow from the requirements

$$\begin{aligned}\varepsilon\left((f,g)_n\right) \;&=\; \varepsilon_f+\varepsilon_g+1+n\,,\\ (f,g)_n = &-(g,f)_n\,(-1)^{(\varepsilon_f+1+n)(\varepsilon_g+1+n)}\,.\end{aligned} \tag{29}$$

The Jacobi identities of the bracket (26), namely

$$((f,g)_n,h)_n\,(-1)^{(\varepsilon_f+1+n)(\varepsilon_h+1+n)} + cycle\,(f,g,h) = 0\,, \tag{30}$$

require in turn

$$E^{IL}\,\partial_L E^{JK}(-1)^{(\varepsilon_I+1+n)(\varepsilon_K+1+n)} + cycle\,(I,J,K) = 0\,. \tag{31}$$

We assume that Z^I span the bracket (26) which implies that E^{IK} is invertible. The inverse, E_{IK}, is defined by

$$E_{IL}E^{LK} = \delta_I^K = E^{KL}E_{LI}\,. \tag{32}$$

It satisfies

$$\begin{aligned}\varepsilon(E_{IK}) = \varepsilon_I+\varepsilon_K+1+n\,, \qquad\qquad & E_{KI} = -E_{IK}(-1)^{(\varepsilon_I+n)(\varepsilon_K+n)+n}\,,\\ \partial_I E_{JK}(Z)(-1)^{(\varepsilon_I+n)\varepsilon_K} + cycle(I,J,K) = 0\,. &\end{aligned} \tag{33}$$

The last relation follows from (32) and the Jacobi identities (31).

We define the generalized antibracket by

$$(F,G) \equiv F \int \frac{\overleftarrow{\delta}}{\delta Z^I(u,\tau)}\, d^n u d^n \tau \left(Z^I(u,\tau), Z^K(u',\tau')\right) d^n u' d^n \tau' \,\frac{\overrightarrow{\delta}}{\delta Z^K(u',\tau')}\, G\,,$$

where (34)

$$\begin{aligned}\left(Z^I(u,\tau), Z^K(u',\tau')\right) &= E^{IK}(Z(u,\tau))(-1)^{n\varepsilon_K}\delta^n(u-u')\,\delta^n(\tau-\tau') \\ &= \delta^n(u-u')\,\delta^n(\tau-\tau')(-1)^{n\varepsilon_I}E^{IK}(Z(u,\tau))\,. \end{aligned} \tag{35}$$

Here the Jacobi identities are

$$\begin{aligned}&\left(Z^I(u,\tau), (Z^J(u',\tau'), Z^K(u'',\tau''))\right)(-1)^{(\varepsilon_I+1)(\varepsilon_K+1)} \\ &+ \mathit{cycle}\,(I,u,\tau;J,u',\tau';K,u'',\tau'') = 0\,, \end{aligned} \tag{36}$$

which are satisfied due to (31). In fact, the properties of E^{IK} imply that the antibracket (34), (35) satisfies all required properties. Inserting (35) into (34) yields the expression

$$(F,G) \equiv F \int \frac{\overleftarrow{\delta}}{\delta Z^I(u,\tau)}\, d^n u\, d^n \tau\, E^{IK}(-1)^{n\varepsilon_K}\frac{\overrightarrow{\delta}}{\delta Z^K(u,\tau)}\, G\,. \tag{37}$$

This antibracket follows also from the relation (6) and the Δ-operator

$$\Delta \equiv \frac{1}{2}\int d^n u\, d^n \tau\, (-1)^{\varepsilon_I}\rho^{-1}\frac{\delta}{\delta Z^I(u,\tau)}\,\rho E^{IK}(Z(u,\tau))(-1)^{n\varepsilon_K}\frac{\delta}{\delta Z^K(u,\tau)}\,, \tag{38}$$

where $\rho(Z(u,\tau))$ is a measure density on the field-antifield space. This Δ-operator differentiates the antibracket (37) according to the rules (8).

A master action Σ in terms of the superfields $Z^I(u,\tau)$ may have, *e.g.* , the form (1) with a Lagrangian density given by

$$\mathcal{L}_n(u,\tau) = V_I(Z(u,\tau))DZ^I(u,\tau)(-1)^{\varepsilon_I+n} - S(Z(u,\tau))\,, \tag{39}$$

where the (anti)symplectic potential $V_I(Z(u,\tau))$ has the Grassmann parity ε_I+n+1. Here we obtain the left functional derivative of Σ in the form

$$\frac{\delta\Sigma}{\delta Z^I} = (-1)^{n\varepsilon_I}\left(E_{IK}DZ^K(-1)^{\varepsilon_K+n} - \partial_I S\right), \tag{40}$$

where E_{IK} is defined by

$$E_{IK}(Z) = \partial_I V_K(Z) - \partial_K V_I(Z)(-1)^{(\varepsilon_I+n)(\varepsilon_K+n)+n}\,. \tag{41}$$

The equations of motion are therefore

$$DZ^I = (S, Z^I)_n \,. \tag{42}$$

Then, consistency requires $(S,S)_n = 0$ again. A still more general form for the master action Σ is obtained from the Lagrangian density

$$\mathcal{L}_n(u,\tau) = Z^K(u,\tau)\,\overline{E}_{KI}(Z(u,\tau))\,DZ^I(u,\tau)(-1)^{\varepsilon_I+n} - S(Z(u,\tau))\,, \tag{43}$$

where

$$\overline{E}_{KI}(Z) \equiv (Z^J\partial_J + 2)^{-1}E_{KI}(Z) = \int_0^1 d\alpha\,\alpha E_{KI}(\alpha Z)\,. \tag{44}$$

(Such an expression was first given in the symplectic case in Ref. 14). Notice also the general form of the expression (18):

$$(\Sigma, Z^I) = (-1)^n \left(DZ^I - (S, Z^I)_n\right). \tag{45}$$

When calculating (Σ,Σ) in terms of the antibracket (34) or (37), where Σ is expressed in terms of (39) or (43), we again find the relation (11). This means that the master equation $(\Sigma,\Sigma) = 0$ also here requires the conditions (13), *i.e.* $\int d^n u d^n \tau D\mathcal{L}_n = 0$, which determines the allowed boundary conditions, and $(S,S)_n = 0$ in terms of the bracket (26) which together with the ghost number prescription determine the allowed form of S.

Here the ghost number prescriptions for Lagrangians (39) and (43) are

$$gh(S) = n, \quad gh(V_I) + gh(Z^I) = n-1, \text{ each } I, \tag{46}$$

and

$$gh(S) = n\,,\; gh(Z^K) + gh(Z^I) + gh(\overline{E}_{IK}) = n-1 \quad (\text{for each } I \text{ and } K)\,, \tag{47}$$

respectively. The relation (47) together with (44), or (46) and (41) imply

$$\begin{aligned} gh(E_{IK}(Z)) &= n - 1 - gh(Z^I) - gh(Z^K), \\ gh(E^{IK}(Z)) &= 1 - n + gh(Z^I) + gh(Z^K), \end{aligned} \tag{48}$$

where the last relation follows from (32). The last equality implies then that the local bracket $(\ ,\ )_n$ carries the extra ghost number $1-n$ (see Eq. (27)), exactly what we had in the previous section. Only for $n = 1$ do we have the standard Poisson bracket.

3 Further generalizations

1. As in Ref. 1, we can choose arbitrary coordinates u^a on the surface. For the measure

$$d^n u\big(\det h_a^b(u)\big)^{-1}\,, \tag{49}$$

we have the nilpotent D-operator

$$D=\tau^a T_a+\frac{1}{2}\tau^b\tau^a U_{ab}^c(u)\frac{\partial}{\partial\tau^c}\ , \tag{50}$$

where

$$\begin{aligned} T_a &\equiv h_a^b(u)\frac{\partial}{\partial u^b}\ ,\\ U_{ab}^c(u) &\equiv -h_a^f(u)h_b^d(u)\frac{\partial}{\partial u^f}\left(h^{-1}\right)_d^c(u)-(a\leftrightarrow b)\,. \end{aligned} \tag{51}$$

Notice that

$$D^2=0\ \Rightarrow\ [T_a,T_b]=U_{ab}^c(u)T_c\,. \tag{52}$$

2. If the original field theory is a superfield theory on a space with coordinates u^a having Grassmann parities ε_a, then the τ^a-coordinates have Grassmann parities ε_a+1. In this case we must assume that the superfields $\Phi^A(u,\tau)$ are still possible to expand as a power series in τ^a which then is an infinite expansion for the bosonic τ^a-coordinates. (The fact that this implies an infinite number of component fields with arbitrarily low ghost numbers suggests that infinite reducibility is a generic feature in the superfield case.) If the original base manifold has dimension (n,m) then the supermanifold has dimension $(n+m,n+m)$ and we get a subdivision into the two cases $n+m$ odd or even. This means that the dimension of the base space still determines the odd and even cases.

3. The local functions S can also have explicit τ-dependence, since the master actions above still satisfy the master equations provided S satisfies (14). However, such master actions are not of the same geometric nature as the previous ones since they do not lead to topological field theories due to the terms with explicit τ-dependence. (This generalization was first considered in the one-dimensional treatments in Refs. 6, 11.)

4. A natural extension of the formalism in Sec. 2 is to let $E^{IK}(Z)$ be singular. It may be, *e.g.* , a Dirac bracket in which case we have

$$E^{IK}_{(\mathcal{D})} \equiv (Z^I, Z^K)_{n(\mathcal{D})} \equiv (Z^I, Z^K)_n - (Z^I, \Theta^\mu(Z))_n C_{\mu\nu} (\Theta^\nu(Z), Z^K)_n \,, \quad (53)$$

where $\Theta^\mu(Z)$ are constraint variables such that $C^{\mu\nu} \equiv (\Theta^\mu, \Theta^\nu)_n$ is invertible with the inverse $C_{\mu\nu}$. Then, according to (35), the basic Dirac antibracket is given by

$$\big(Z^I(u,\tau), Z^K(u',\tau')\big)_{(\mathcal{D})} = E^{IK}_{(\mathcal{D})}(Z(u,\tau))\,(-1)^{n\varepsilon_K}\,\delta^n(u-u')\,\delta^n(\tau-\tau')\,. \quad (54)$$

The local action S should be replaced by

$$S(Z(u,\tau)) \;\longrightarrow\; S(Z(u,\tau)) + \Pi_\mu(u,\tau)\,\Theta^\mu(Z(u,\tau))\,, \quad (55)$$

where Π_μ is a Lagrange multiplier superfield.

4 Generalized Poisson sigma models in any dimension

Consider dimension n and consider first superfield pairs X^i and X^*_i with the properties

$$\begin{aligned} \varepsilon(X^*_i) &= \varepsilon_i + 1 + n\,, \qquad\qquad \varepsilon_i \equiv \varepsilon(X^i)\,, \\ gh(X^*_i) &= n - 1 - gh(X^i)\,, \end{aligned} \quad (56)$$

which are required by our general rules in Sec. 1. Let us now consider the local function

$$S = \frac{1}{2} X^*_j X^*_i \omega^{ij}(X)(-1)^{\varepsilon_j + n}. \quad (57)$$

From the general prescription

$$\varepsilon(S) = n\,, \quad gh(S) = n\,, \quad (58)$$

it follows that the functions $\omega^{ij}(X)$ must satisfy the properties

$$\begin{aligned} \varepsilon(\omega^{ij}) &= n + \varepsilon_i + \varepsilon_j\,, \\ gh(\omega^{ij}) &= 2 - n + gh(X^i) + gh(X^j)\,. \end{aligned} \quad (59)$$

The expression (57) implies furthermore that

$$\omega^{ij} = -\,\omega^{ji}(-1)^{(\varepsilon_i+n)(\varepsilon_j+n)}. \quad (60)$$

The master equation $(S,S)_n = 0$ requires then

$$\omega^{il}\partial_l\omega^{jk}(-1)^{(\varepsilon_i+n)(\varepsilon_k+n)} + cycle\,(i,j,k) = 0\,. \quad (61)$$

From these results it follows that $\omega^{ij}(X)$ has exactly the same properties as the local bracket (26) for $n-1$. Therefore, we can make the identification

$$\omega^{ij}(X) = (X^i, X^j)_{n-1}\,, \tag{62}$$

where $(\ ,\ )_0$ can be identified with the conventional antibracket in the field-antifield formalism. For even n ω^{ij} is a Poisson bracket and for odd n ω^{ij} is an antibracket. But only for $n = 1, 2$ do these brackets have the conventional ghost number prescriptions.[a] The boundary conditions must be consistent with (13).

When $\omega^{ij}(X)$ satisfies the above properties and when the boundary conditions are consistent with (13) then the action

$$\Sigma[X, X^*] = \int d^n u d^n \tau \left(X_i^* D X^i (-1)^{\varepsilon_i + n} - S \right) \tag{63}$$

satisfies the master equation $(\Sigma, \Sigma) = 0$. However, for this case the original models can only be written down after we have given a more explicit form for $\omega^{ij}(X)$. The only exception is for $n = 2$ and $gh(X^i) = 0$ in which case, according to the rules (25), we directly obtain

$$A = \int \Big(x_i^* \wedge dx^i - \frac{1}{2} x_j^* \wedge x_i^* \omega^{ij}(x) \Big)\,, \tag{64}$$

where x_i^* and x^i are one-form and zero-form fields respectively. (The Grassmann parities are $\varepsilon(x_i^*) = \varepsilon(x^i) \equiv \varepsilon_i$.) This is just the well-known Poisson sigma model[2,3] for which Cattaneo and Felder also gave the superfield master action[4] given here for $\varepsilon(x^i) = 0$. One can easily check that the boundary conditions[4] are consistent with (13).

In Ref. 1 generalized Poisson sigma models for $n=2$ were constructed by means of the algorithm presented here. This construction can also be generalized to arbitrary n. We consider then the following expression for S:

$$S = \frac{1}{2} X_j^* X_i^* \omega^{ij}(X)(-1)^{\varepsilon_j + n} + \Lambda_\alpha^* \theta^\alpha(X)\,, \tag{65}$$

where we have introduced new superfield pairs Λ_α^*, Λ^α with the properties

$$\begin{aligned} &\varepsilon(\Lambda_\alpha^*) = \varepsilon_\alpha + n\,, \qquad &&\varepsilon(\Lambda^\alpha) = \varepsilon_\alpha + 1\,, \qquad &&\varepsilon_\alpha \equiv \varepsilon(\theta^\alpha)\,, \\ &gh(\Lambda_\alpha^*) = n - gh(\theta^\alpha)\,, \quad &&gh(\Lambda^\alpha) = gh(\theta^\alpha) - 1\,. \end{aligned} \tag{66}$$

[a] $(\ ,\ )_1$ is the conventional Poisson bracket.

Apart from (61), the local master equation $(S,S)_n = 0$ requires now that

$$\theta^\alpha(X)\overleftarrow{\partial}_j \omega^{ji}(X) = 0\,, \tag{67}$$

which means that $\omega^{ji}(X)$ can be interpreted as a Dirac bracket of the $n-1$ type, *i.e.* a bracket $(\cdot,\cdot)_{n-1}$ satisfying $(\cdot,\theta^\alpha(X))_{n-1} = 0$.

The expression (65) is not the general expression of S in terms of these fields. We can also add the terms

$$\begin{aligned}(-1)^{\varepsilon_j+n+(\varepsilon_i+n)(\varepsilon_k+n)}\,\frac{1}{6}X^*_i X^*_j X^*_k \omega^{kji}_\alpha(X)\Lambda^\alpha \\ +(-1)^{\varepsilon_i+n}X^*_i\Lambda^*_\alpha\omega^{\alpha i}_\beta(X)\Lambda^\beta+\ldots\,,\end{aligned} \tag{68}$$

where

$$\begin{aligned}\varepsilon(\omega^{kji}_\alpha(X)) &= \varepsilon_i+\varepsilon_j+\varepsilon_k+\varepsilon_\alpha\,,\\ \varepsilon(\omega^{\alpha i}_\beta(X)) &= \varepsilon_i+\varepsilon_\alpha+\varepsilon_\beta+n\,,\\ gh(\omega^{kji}_\alpha(X)) &= gh(X^i)+gh(X^j)+gh(X^k)-gh(\theta^\alpha)+4-2n\,,\\ gh(\omega^{\alpha i}_\beta(X)) &= gh(X^i)+gh(\theta^\alpha)-gh(\theta^\beta)+2-n\,,\end{aligned} \tag{69}$$

and where ω^{kji}_α is totally antisymmetric in i, j, k with Grassmann parity $\varepsilon_{ijk} = (\varepsilon_i+n)(\varepsilon_j+n)+(\varepsilon_j+n)(\varepsilon_k+n)+(\varepsilon_k+n)(\varepsilon_i+n)$. In this case the master equations yield a weak form of the Jacobi identities (61) and the degeneracy condition (67), which means that $\omega^{ij}(X)$ now is a weak Dirac bracket of the $n-1$ type. For all these forms of S the master action is given by

$$\Sigma[X,X^*,\Lambda,\Lambda^*]=\int d^nud^n\tau\Big(X^*_iDX^i(-1)^{\varepsilon_i+n}+\Lambda^*_\alpha D\Lambda^\alpha(-1)^{\varepsilon_\alpha+n+1}-S\Big). \tag{70}$$

We can also add further superfield pairs Ξ^{α_k}, $\Xi^*_{\alpha_k}$, $k=1,\ldots,L$, which enter S to leading order in the form

$$\sum_{k=0}^{L-1}\Xi^*_{\alpha_{k+1}}Z^{\alpha_{k+1}}_{\alpha_k}(X)\,\Xi^{\alpha_k}\,,\qquad (k\geq 0\,,\quad \alpha_0\equiv\alpha\,,\quad \Xi^{\alpha_0}\equiv\Lambda^\alpha)\,,$$

where

$$\begin{aligned}\varepsilon(Z^{\alpha_{k+1}}_{\alpha_k}) &= \varepsilon_{\alpha_{k+1}}+\varepsilon_{\alpha_k}\,,\\ \varepsilon(\Xi^{\alpha_k}) &\equiv \varepsilon_{\alpha_k}+k+1\,,\\ gh(Z^{\alpha_{k+1}}_{\alpha_k}) &= gh(\Xi^{\alpha_{k+1}})-gh(\Xi^{\alpha_k})+1\,.\end{aligned} \tag{71}$$

Here the master action is

$$\Sigma\left[X, X^*, \Xi, \Xi^*\right] = \tag{72}$$

$$= \int d^n u d^n \tau \Big(X_i^* D X^i (-1)^{\varepsilon_i + n} + \sum_{k=0}^{L} \Xi^*_{\alpha_k} D\, \Xi^{\alpha_k} (-1)^{\varepsilon_{\alpha_k} + k + 1 + n} - S \Big) ,$$

and the master equations tell us that θ^α is reducible to stage L, which means $Z^{\alpha_1}_{\alpha}\theta^\alpha = 0\,,\ Z^{\alpha_3}_{\alpha_2} Z^{\alpha_2}_{\alpha_1} \approx 0\,,\ \ldots,\ Z^{\alpha_L}_{\alpha_{L-1}} Z^{\alpha_{L-1}}_{\alpha_{L-2}} \approx 0\,.$

The original model and its properties of all these modified forms of S can only be written down when we have given a more explicit form for the ghost dependent functions. However, for $n = 2$ and $gh(X^i) = 0$ we obtain, according to the rules (25), for all the above cases the original action,

$$A = \int \Big(x_i^* \wedge dx^i - \frac{1}{2}\, x_j^* \wedge x_i^* \omega^{ij}(x) - \lambda_\alpha \theta^\alpha(x) \Big) , \tag{73}$$

where λ_α is a Lagrange multiplier 2-form field.

Still more generalized forms of Poisson sigma models might be possible to derive if we allow S to have explicit τ-dependence which is possible according to the generalization 3 in Sec. 3.

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Appendix A: Further applications

Models in $n = 1$

Consider first superfield pairs X^i and X_i^* with ghost number zero and arbitrary Grassmann parities $\varepsilon(X_i^*) = \varepsilon(X^i)$. If these are the only superfields then $S = 0$, since S must have ghost number one. The original model is trivial then and is of the form $A = \int x_i^* dx^i$. If we also have the superfield pairs Λ^*_α and Λ^α with ghost numbers one and minus one respectively, then S has the general form $(\varepsilon(\Lambda^*_\alpha) = \varepsilon(\Lambda^\alpha) = \varepsilon_\alpha + 1,\ \varepsilon_\alpha \equiv \varepsilon(\theta^\alpha))$

$$S = \Lambda^*_\alpha \theta^\alpha(X, X^*) + \frac{1}{2}\Lambda^*_\alpha \Lambda^*_\beta U^{\beta\alpha}_\gamma(X, X^*)\Lambda^\gamma(-1)^{\varepsilon_\alpha} + \ldots\ , \tag{A.1}$$

where the dotted terms are determined by the condition $(S, S)_1 = 0$. Notice that S is odd and that $(\ ,\ )_1$ is a conventional Poisson bracket. In fact,

$$(S,S)_1 = 0 \quad \Rightarrow \quad (\theta^\alpha, \theta^\beta)_1 = U_\gamma^{\alpha\beta}\theta^\gamma\,. \tag{A.2}$$

Thus, the master action

$$\Sigma\big[X, X^*, \Lambda, \Lambda^*\big] = \int du\, d\tau\, \big(X_i^* DX^i(-1)^{\varepsilon_i+1} + \Lambda_\alpha^* D\Lambda^\alpha(-1)^{\varepsilon_\alpha} - S\big)\,, \tag{A.3}$$

where S is defined by (A.1), satisfies the master equations provided the constraint variables θ^α are in involutions. We can also add further pairs $\Xi_{\alpha_k}^*$, Ξ^{α_k} with ghost numbers $k > 1$ and $-k$. The resulting S satisfying $(S,S)_1 = 0$ will then imply that the constraint variables θ^α are reducible (linearly dependent) up to a certain stage. Thus, S may attain the general form of a BFV-BRST charge for a constraint theory where the constraints are in arbitrary involutions on the phase space spanned by the canonical coordinates X^i and X_i^*. Λ_α^* are ghosts and $\Xi_{\alpha_k}^*$ ghost for ghosts. The corresponding BRST charge for the original model is the $\tau = 0$ component of the $S(u,\tau)$ in (A.1). The boundary condition (13) requires the conservation of this charge, *i.e.* $S(u_2, 0) - S(u_1, 0) = 0$ where u_1 and u_2 are the limits of the u-integration in Σ. The original action is obtained from the corresponding master action using the rules (25). We find for all the above cases the original action

$$A = \int \big(x_i^* dx^i - \lambda_\alpha \theta^\alpha(x, x^*)\big)\,, \tag{A.4}$$

where λ_α is a Lagrange multiplier one-form field. This action is defined on the phase space where x_i^* and x^i are canonical conjugate variables (x_i^* are conjugate momenta to x^i). It is gauge invariant under the gauge transformations

$$\begin{aligned}\delta x^i &= (x^i, \theta^\alpha)_1\beta_\alpha, \quad \delta x_i^* = (x_i^*, \theta^\alpha)_1\beta_\alpha,\\ \delta\lambda_\alpha &= d\beta_\alpha(-1)^{\varepsilon_\alpha} - \beta_\mu\lambda_\nu U_\alpha^{\nu\mu}(-1)^{\varepsilon_\mu},\end{aligned} \tag{A.5}$$

where β^α are the gauge parameters. The theory (A.4) is an arbitrary constraint theory with zero Hamiltonian, which implies that the theory is reparametrization invariant. In fact, any one-dimensional Hamiltonian model can be cast into this form, since any such theory can be cast into a reparametrization invariant form. However, a nonzero Hamiltonian $H(X, X^*)$ is possible to introduce if we allow S to have an explicit τ-dependence (generalization 3 in Sec. 3, see also Ref. 6). Condition (14) requires then $H(X, X^*)$ to have zero local Poisson bracket with the S in (A.1). We can also choose fields which are arbitrary symplectic coordinates if we make use of the general master actions following from (39) and (43). These results agree with those obtained in Ref. 6.

The Chern-Simon model and generalizations

In $n = 3$ the natural choice of superfield pairs are X^i and X^*_j both with ghost number one. Since S must have ghost number three, S must be trilinear in X^i and X^*_j. This case is simpler to analyze if we let the superfield Z^I represent both X^i and X^*_j. Then, Z^I are Darboux coordinates on a symplectic manifold. The local Poisson bracket is then

$$(Z^I, Z^K)_3 = E^{IK}\,, \tag{A.6}$$

where E^{IK} is constant. (Notice that the bracket $(\ ,\ )_3$ carries an extra ghost number -2 which implies $gh(E^{IK}) = 0$ for $gh(Z^I) = 1$). The general form of S is in this case

$$S = \frac{1}{6}\,C_{IJK}Z^IZ^JZ^K\,. \tag{A.7}$$

If we let the fields Z^I be odd fields corresponding to original fields which are even one-form fields, then C_{IJK} in (A.7) are even real constants. Notice also that E^{IK} in (A.6) is symmetric for odd fields. In this case the master action with the Lagrangian density (39) becomes

$$\Sigma[Z] = \frac{1}{2}\int d^3u\, d^3\tau \Big(E_{IK}Z^KDZ^I - \frac{1}{3}\,C_{IJK}Z^IZ^JZ^K\Big)\,. \tag{A.8}$$

The master equation $(\Sigma, \Sigma) = 0$ allows us to interpret C_{IJK} as structure coefficients of a Lie group, since C_{IJK} is totally antisymmetric from (A.7) and since $(S, S)_3 = 0$ requires them to satisfy the Jacobi identities. E^{IK} acts as a group metric. According to the rules (25), the original model is

$$A = \frac{1}{2}\int E_{IK}z^I \wedge \Big(dz^K - \frac{1}{3}\,E^{KL}C_{LMN}\,z^M{\wedge}z^N\Big)\,, \tag{A.9}$$

which is a non-abelian Chern-Simon model, z^I being even, one-form fields. The Chern-Simon model was also treated in Ref. 5.

We can also introduce superfield pairs with ghost numbers two and zero respectively, and pairs with ghost numbers three and minus one. These superfields allow for many more terms in S which are consistent with the ghost number prescription. It seems likely that the local master equation (14) will allow for new nontrivial solutions, in which case one will find generalized Chern-Simon models. All fields must satisfy boundary conditions which are consistent with (13).

For $n = 4$ we can obtain something similar to the Chern-Simon model if we choose superfield pairs X^i and X^*_i with ghost number one and two, respectively.

It is natural then to let X^i be odd and X_i^* be even since the corresponding original one- and two-form fields, x^i and x_i^*, then are even according to the rule (25). The general form of S is then

$$S = \frac{1}{2} C^{ij} X_i^* X_j^* + \frac{1}{2} C^i{}_{jk} X_i^* X^j X^k + \frac{1}{24} C_{ijkl} X^i X^j X^k X^l \,, \tag{A.10}$$

where the coefficients are even real constants. In this case the condition $(S,S)_4 = 0$ yields the following three conditions:

$$C^i{}_{jk} C^{kl} X_i^* X_l^* X^j = 0 \,, \tag{A.11}$$

$$\Big(C^i{}_{jm} C^m{}_{kl} + \frac{1}{3} C_{jklm} C^{mi} \Big) X_i^* X^j X^k X^l = 0 \,, \tag{A.12}$$

$$C_{ijkn} C^n{}_{lm} X^i X^j X^k X^l X^m = 0 \,. \tag{A.13}$$

We notice the following special solutions:

i) If $C^{ij} = C_{ijkl} = 0$ then $C^i{}_{jk}$ can be interpreted as structure coefficients of a Lie group since $(S,S)_4 = 0$ requires the Jacobi identities due to (A.12).

ii) If only $C_{ijkl} = 0$, then the same interpretation of $C^i{}_{jk}$ is possible. However, in this case $(S,S)_4 = 0$ requires not only the Jacobi identities for $C^i{}_{jk}$ but also the condition

$$C^i_{jl} C^{lk} + C^k_{jl} C^{li} = 0 \,, \tag{A.14}$$

from (A.11). If C^{ij} is an invertible matrix, then C_{ij} exists satisfying $C^{ij} C_{jk} = \delta^i_k$ and (A.14) can be rewritten as

$$C_{ijk} + C_{kji} = 0 \,, \qquad C_{ijk} \equiv C_{il} C^l{}_{jk} \,. \tag{A.15}$$

Thus, the symmetric matrix C^{ij} can be interpreted as a group metric and C_{ijk} is totally antisymmetric as required by a semi-simple Lie group. This corresponds to what we had in the Chern-Simon model in three dimensions. According to the rules (25), the original model is

$$A = \int \Big(x_i^* \wedge dx^i - \frac{1}{2} C^{ij} x_i^* \wedge x_j^* - \frac{1}{2} C^i{}_{jk} x_i^* \wedge x^j \wedge x^k \Big) \,, \tag{A.16}$$

where x^i and x_i^* are even one- and two-form fields, respectively. (They are the ghost number zero coefficients of X^i and X_i^*.) This action is gauge invariant under the transformations

$$\begin{aligned} \delta x^i &= d\beta^i + C^{ij} \gamma_j + C^i{}_{jk} \beta^j x^k \,, \\ \delta x_i^* &= d\gamma_i + C^k_{ij} x_k^* \beta^j + C^k_{ij} \gamma_k \wedge x^j \,, \end{aligned} \tag{A.17}$$

where the gauge parameters β^i and γ_i are zero- and one-forms, respectively.

Further, we can also introduce superfield pairs here, with ghost numbers three and zero respectively and pairs with ghost numbers four and minus one, etc, which will allow for new terms in S. The local master equation (14) should then allow for new nontrivial solutions in which case one obtains generalized models. Again, all fields must satisfy boundary conditions which are consistent with (13).

From the kinetic terms in the superfield Lagrangians (2) it is clear that we get a BF-theory in any dimension n, and in any dimension we can also have a Lagrangian multiplier which is an n-form field in the original actions. However, in higher and higher dimensions there are more and more different k-form fields allowed. The most general structure will therefore be more and more complex in higher and higher dimensions.

Appendix B: Superfields in terms of component fields

A decomposition of the superfields in component fields satisfying (19) can be obtained by means of the following recursive formula. We start with zero components Φ_0^A and Φ^*_{0B} satisfying

$$(\Phi_0^A(u), \Phi^*_{0B}(u')) = \delta^A_B \delta^n(u-u') \,. \tag{B.1}$$

We write then the superfields as follows

$$\Phi^A(u,\tau^1,\ldots,\tau^{n-1},\tau^n) = \Phi_0^A(u,\tau^1,\ldots,\tau^{n-1}) + \tau^n \Phi_1^A(u,\tau^1,\ldots,\tau^{n-1}) \,, \tag{B.2}$$

$$\begin{aligned}\Phi^*_A(u,\tau^1,\ldots,\tau^{n-1},\tau^n) &= \\ &= \Big((-1)^{n-1}\Phi^*_{1A}(u,\tau^1,\ldots,\tau^{n-1}) - \Phi^*_{0A}(u,\tau^1,\ldots,\tau^{n-1})\tau^n\Big)(-1)^{\varepsilon_A} \,,\end{aligned} \tag{B.3}$$

where
$$\begin{aligned}&\big(\Phi^A(u,\tau^1,\ldots,\tau^{n-1}), \Phi^*_B(u',\tau'^1,\ldots,\tau'^{n-1})\big) \\ &\qquad = \delta^n(u-u')\delta^{n-1}(\tau-\tau')\delta^A_B(-1)^{(n-1)\varepsilon_A} \,.\end{aligned} \tag{B.4}$$

Then the n-parametric superfields (B.2) and (B.3) satisfy by construction the relations (B.4) with the formal replacement $n \to n+1$. One can easily check that the following expressions satisfy these recursion formulas:

$$\begin{aligned}\Phi^A(u,\tau) &= \Phi_0^A(u) + \tau^a \Phi_a^A(u) + \frac{1}{2}\tau^{a_1}\tau^{a_2}\Phi^A_{a_1a_2}(u) + \cdots \\ &\quad \cdots + \frac{1}{k!}\tau^{a_1}\cdots\tau^{a_k}\Phi^A_{a_1\cdots a_k}(u) + \cdots \\ &\quad \cdots + \frac{1}{n!}\tau^{a_1}\cdots\tau^{a_n}\Phi^A_{a_1\cdots a_n}(u) \,,\end{aligned} \tag{B.5}$$

$$\begin{aligned}\Phi^*_A(u,\tau) = \bigg(&\frac{1}{n!}\,\Phi^{*\,a_1\cdots a_n}_A(u) - \frac{1}{(n-1)!}\,\Phi^{*\,a_1\cdots a_{n-1}}_A(u)\,\tau^{a_n} \\ &+ \frac{1}{2}\,\frac{1}{(n-2)!}\,\Phi^{*\,a_1\cdots a_{n-2}}_A(u)\,\tau^{a_{n-1}}\tau^{a_n} + \cdots \\ &\cdots + (-1)^k\frac{1}{k!(n-k)!}\,\Phi^{*\,a_1\cdots a_{n-k}}_A(u)\,\tau^{a_{n-k+1}}\cdots\tau^{a_n} + \cdots \\ &\cdots + (-1)^n\frac{1}{n!}\,\Phi^{*\,0}_A(u)\,\tau^{a_1}\cdots\tau^{a_n}\bigg)(-1)^{n\varepsilon_A}\varepsilon_{a_1\cdots a_n}\,, \end{aligned} \tag{B.6}$$

where $\varepsilon_{a_1a_2\cdots a_n}$ is totally antisymmetric such that $\varepsilon_{12\cdots n}=1$. We also define $\varepsilon_1\equiv 1$, $\varepsilon_0\equiv 1$. Notice that $\Phi^A_{a_1\cdots a_k}$ and $\Phi^{*a_1\cdots a_k}_A$ are totally antisymmetric, and that we have

$$(\Phi^A_{a_1\cdots a_k}(u), \Phi^{*\,a_1\cdots a_k}_B(u')) = \delta^A_B\,\delta^n(u-u')\,. \tag{B.7}$$

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