PARTICLE PRODUCTION IN COSMOLOGY AND IMAGINARY TIME METHOD

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After brief personal recollections of the author's long-time friendship with Misha Marinov the problem of particle production by classical time-varying scalar field is discussed. In the quasiclassical limit the calculations are done by imaginary time method developed, in particular, in Marinov's works. The method permits to obtain simple analytical expressions which well agree with the later found numerical solutions. The results are compared with perturbative calculations and it is argued that perturbation theory gives an upper limit for the rate of production.

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1 Personal recollections

It is difficult to write about a close friend who died so unexpectedly and so early. Time runs fast and it seems that it was just yesterday that we saw each other and talked a lot on every possible subject. I really miss these discussions now. The last time that I saw Misha Marinov was spring 1999. I was visiting Weizmann Institute of Science in the frameworks of the Landau-Weizmann Program and used this opportunity to come to Haifa where Misha lived and worked in Technion. It was a nice sunny and fresh morning when we arrived with my wife Inna at the Haifa railway station, where Misha met us and drove along a beautiful road up the Carmel Mountain to his home where Lilia, his wife, waited us with a delicious lunch. Misha was in high spirits, the four of us being old friends, and we were very glad to see each other, although he slightly complained about, as he said, a small pain in his spine. None of us knew at that moment that it was a first sign of a fast and terrible disease.

Our friendship with Misha began, I think, in 1965 when we both were graduate students. We happened to be in the same plane on the way to Yerevan to the First International Nor-Ambert School on Particle Physics. Together with another young physicist from ITEP, Misha Terentev, we shared a small hut and enjoyed our first international conference and the charming and hospitable town of Yerevan.

Misha was two years older than me but I had a feeling that his knowledge of physics, especially of mathematical physics, was at a professor's level, and benefited a lot from our communications. Later we both worked at ITEP theory group and it was always instructive and interesting to talk with him not only about physics but about practically any subject, especially history where Misha had unusually deep and extensive knowledge whether it be ancient or modern.

Our friendship turned into friendship between families when in 1970 we started to live in the same apartment building near ITEP and the distance between our apartments was only 1-2 minute walk up or down the stairs. In 1979 Misha quit his position at ITEP and applied for permission to emigrate to Israel. Immediately life became much harder for him. It was difficult to find a job that could give enough money to support his family of four. Special rules existed at that time in the Soviet Union to prevent people from working without strict state control. In summer seasons (plus a part of spring and autumn) Misha worked as a construction worker, building small private houses (dachas) in the country. In winter he did some work for the official "Center of Translations" translating scientific papers or books from English into Russian or vice versa. However this kind of job was allowed only if one has another permanent place of work at one or another state enterprise, which Misha had not. At some stage they requested from him a certificate that he had such a job and since nothing could be presented, he was fired. So my wife, Inna, formally took this job and fetched for him papers to translate from the Center. Misha translated them, Inna presented the translated papers to the Center, received the money and brought it to Misha or his wife, Lilia. That's how it worked.

I have to confess that I also participated in a similar deceptive activity. A few papers and books translated into Russian under my name were in fact translated by Misha. In particular, the review paper by S. Coleman "The magnetic monopole fifty years later"[1] was translated for Uspekhi Fiz. Nauk, vol. 144, by Marinov but under my name. Moreover, the editors wanted to have a short review on the activity related to magnetic monopoles up until the moment when the paper was translated, i.e. two years after the original one had been written. Again, Misha wrote the paper and I only signed it (honestly, I also read it and liked it very much). So he got the money and I got the fame. Now I have to set things right and change Ref. 2 into Ref. 3.

Only in 1987 the Marinovs received permission to emigrate and left for Israel. As we all thought that time, emigration meant leaving for good with practically zero chances to see or contact each other again. However, things were changing fast and freedom to travel abroad, unbelievable during Soviet times, came to our country. In 1990 Inna was able to go to Israel and for a whole month enjoyed friendly atmosphere of Marinov's home. She and Lilia even now recall with mutual pleasure how nice that time was. After a couple of years Misha's life in Israel was successfully arranged. He got a professor position in Technion and was happy to be there. I remember how proudly he showed me the campus, labs and students during my first visit to Haifa. He enthusiastically returned to research that had been interrupted for 6-7 years. As I can judge by what and how I learned from him, he was a very good teacher and did teaching with vigor and love. On the other hand, there remained warm feelings toward ITEP, and often, when going in the morning to his office in Technion, he used to say, addressing Inna and Lilia, "Bye girls, I am going to ITEP."

There are several fields where Misha made very important contributions, despite a long break in his scientific activity. But I am not going to describe them all, since, I think, this will be described in the Introduction to this volume. I will mention only two which have some relation to me. Misha's results on the application of path integral methods to complicated quantum systems are internationally renowned and I am proud that I recommended Maurice Jacob to publish Marinov's review on the subject in Physics Reports.[4] This was the last paper written by Misha, while he was still in ITEP. Another subject where M. Marinov made a very essential contribution together with V. Popov was electron-positron production by an external electromagnetic field.[5] The method developed in these works was applied to the non-perturbative calculations of cosmological particle production by scalar (inflaton) field in our paper with D. Kirilova,[6] which is discussed below.

2 Particle production in cosmology: brief historical review

There are two different cases of quantum particle production by external classical fields that are cosmologically interesting. The first is the production by time-dependent background metric or, in other words, by gravitational field and the second is the transformation of classical (oscillating) inflaton field into elementary particles and the corresponding universe (re)heating. Particle production by gravity might be essential in the very early universe near cosmological singularity when the strength of gravitational field was close to the Planck value. Creation of particles by isotropic Friedman-Robertson-Walker (FRW) metric was pioneered by Parker[7] and further developed in a series of papers.[8,9,10] Particle production by gravity in anisotropic cosmologies was considered in Refs. 11. As argued in these papers, particle production in anisotropic case creates anisotropic distribution of matter and back reaction of the created matter on the metric could lead to isotropization of the latter. Thus, in principle, the observed FRW cosmology might originate from a rather general initial state. More references to the subsequent works and detailed discussion can be found in the books.[12]

There is an important difference between particle production in isotropic and anisotropic cosmologies. Isotropic FRW metric is known to be conformally flat, i.e. after a suitable coordinate transformation it can be reduced to the form:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = a^2(r,\tau)\left(d\tau^2 - d\vec{r}^{\,2}\right). \tag{1}$$

It follows from this expression, in particular, that FRW metric cannot create massless particles if the latter are described by conformally invariant theory.[7] If the particle mass m is non-vanishing but the interactions are conformally invariant, their production rate is suppressed as a power of the ratio (m/m_{Pl}). (Of course, non-vanishing masses break conformal invariance.) These statements can be easily checked in perturbation theory. The coupling of gravity to matter fields is given by $(g^{\mu\nu} - \eta_{\mu\nu})\, T^{\mu\nu}$, where $\eta^{\mu\nu}$ is the Minkowski metric tensor and $T^{\mu\nu}$ is the energy-momentum tensor of matter. If the metric tensor is given by expression (1), the coupling to matter is proportional to the trace of the energy-momentum tensor that vanishes in conformally invariant theory.

A well known example of the theory which is conformally invariant at classical level (i.e. without quantum corrections) is electrodynamics with massless charged fermions, or any other (possibly non-abelian) gauge theory describing interacting massless gauge bosons and fermions. However quantum trace anomaly[13] breaks conformal invariance and gives rise to a non-zero trace of $T_{\mu\nu}$. In SU(N) gauge theory with N_f number of fermions the trace of the energy-momentum tensor of matter is equal to

$$T^{\mu}_{\mu} = \frac{\alpha}{\pi}\left(\frac{11N}{3} - \frac{2N_f}{3}\right) G_{\mu\nu}G^{\mu\nu}\,, \tag{2}$$

where $G_{\mu\nu}$ is the gauge field strength tensor. This anomaly could strongly enhance generation of electromagnetic field (or any other gauge fields) in the early universe.[14]

Another simple and important theory of a free massless scalar field ϕ is not conformally invariant even at the classical level if ϕ is minimally coupled to gravity (that is through covariant derivatives only). The energy-momentum tensor of such field is given by

$$T_{\mu\nu}(\phi) = \frac{1}{2}\,\partial_\mu\phi\,\partial_\nu\phi - \frac{1}{4}\,g_{\mu\nu}\partial_\alpha\phi\,\partial^\alpha\phi \tag{3}$$

and its trace $T^{\mu}_{\mu} = -(1/2)\partial_\alpha\phi\,\partial^\alpha\phi$ is generally non-vanishing. Conformal invariance can be restored if one adds to the free Lagrangian the nonminimal coupling to gravity in the form $R\phi^2/12$ (see e.g. Refs. 12). However it would be better not to restore it because generation of primordial density perturbations at inflation,[15] which serve as seeds for large scale structure formation, is possible only for non-conformal fields.

Another realistic example of conformally non-invariant theory with massless fields is gravity itself. It was shown that gravitational waves are not conformally invariant in the standard General Relativity.[16] This explains efficient production of gravitational waves during inflationary stage.[17]

A renewed interest to gravitational particle production arose in connection with a possible explanation of the observed ultra-high energy cosmic rays by heavy particle decays.[18] There are two competing mechanisms of creation of such particles in cosmology: by background metric and by inflaton field. The former was considered in Refs. 19 (for a review see Ref. 20), while particle production by inflaton will be discussed below.

In the earlier papers[21] the universe (re)heating at the final stage of inflation through particle production by the oscillating inflaton field was treated in a simplified perturbation theory approximation. First non-perturbative treatment was performed in two papers.[6,22] In what follows we concentrate on the approach of Ref. 6 where the imaginary time method was used. In both papers[6,22] a possibility of parametric resonance enhancement of particle production rate, noticed long ago,[23] was mentioned. However, it was argued in the first of them that the resonance was not effective because the produced particles were quickly removed from the resonance band by the cosmological expansion and elastic scattering on the background. A more careful analysis of the subsequent paper[22] showed that under certain condition expansion might be irrelevant and did not destroy the resonance. In this case a strong amplification of the production probability and much faster process of post-inflationary (re)heating could be expected. After Ref. 24 the issue of the parametric resonance (re)heating has attracted great attention, and now the number of published papers on the subject is measured by a few hundreds. However, a review of this activity is outside the scope of the present paper and below we will confine ourselves to the problem of fermion production by a time dependent scalar field where parametric resonance is not effective.

Concerning production of fermions, there is a contradiction in the literature between the paper,[6] where non-perturbative production of fermions was pioneered, and the subsequent ones. While in Ref. 6 it was stated that fermion production is always the strongest in perturbation theory regime, and in the opposite – quasiclassical – limit the production is noticeably weaker, in subsequent works it was argued that in non-perturbative regime fermion production was strongly enhanced so that it could even compete with resonant boson production. Calculations in Ref. 6 have been performed by imaginary time method, while other works used either numerical calculations or some approximate analytical estimates. I will argue in what follows that there is practically no difference between the results of all calculations, earlier and later ones, but the difference is in the interpretation of the results and that fermion production by the inflaton is always weak, weaker than that found in perturbation theory.

3 Particle production in perturbation theory

Let us start from consideration of production in the case when perturbation theory is applicable and calculations are straightforward and simple. In this section we will neglect the universe expansion and assume that the external scalar field periodically changes with time according to

$$\phi(t) = \phi_0 \cos \omega t \,. \tag{4}$$

Here ϕ_0 is the amplitude of the field, it can be slowly varying function of time, and the frequency of oscillations ω coincides with the mass of ϕ if the latter lives in the harmonic potential $U(\phi) = m_\phi^2 \phi^2/2$.

We assume that ϕ is coupled to fermions through the Yukawa interaction,

$$\mathcal{L}_\psi = \bar\psi \left(i\partial\!\!\!/ + m_0\right)\psi \, + \, g\phi\bar\psi\psi \,. \tag{5}$$

Perturbation theory would be valid if the coupling constant were small, $g \ll 1$, which is well fulfilled for the inflaton field, and if the fermion mass is smaller than the mass of the inflaton, $m_\phi = \omega$. The last condition may not be true even if $m_0 < m_\phi$ because the interaction with ϕ introduces effective time-dependent mass

$$m_1(t) = g\phi_0 \cos\omega t \,, \tag{6}$$

and for a large amplitude ϕ_0 the latter may be large in comparison with ω for most of the oscillation period, except for a small part, when $\cos\omega t$ is close to zero. In this case perturbation theory is invalid.

It is practically evident, even without calculations, that in perturbative case the rate of particle production is equal to the width of the decay of the scalar boson ϕ into a pair of fermions:

$$\dot n_\psi / n_\phi = \Gamma_\phi = g^2\omega/8\pi \,, \tag{7}$$

where $n_{\psi,\phi}$ are the number densities of ψ and ϕ particles per unit volume respectively and we assumed for simplicity that the fermion mass $m_0 = 0$ (it is straightforward to lift this restriction).

Still, to make a comparison with subsequent non-perturbative calculations, we will sketch below the derivation of this result. According to general rules of quantum field theory, the amplitude of production of a pair of particles with momenta $\vec p_1$ and $\vec p_2$ by an external time-dependent field $\phi(t)$ in the first order of perturbation theory is given by

$$A(\vec p_1, \vec p_2) = g \int d^4x \;\phi(t)\, \langle \vec p_1, \vec p_2 |\, \bar\psi(x)\psi(x)\, | vac\rangle \,, \tag{8}$$

where the state $\langle \vec p_1, \vec p_2|$ is produced by action of the creation operators on vacuum in the standard second-quantized decomposition of Dirac operators ψ and $\bar\psi$:

$$\psi(x) = \sum_s \int \frac{d^3k}{(2\pi)^3} \left[u^s_k b^s_k e^{-ik\cdot x} + v^s_k d^{s\dagger}_k e^{ik\cdot x} \right] , \tag{9}$$

where b^s_k and $d^{s\dagger}_k$ are annihilation and creation operators for particles and antiparticles with momentum k and spin s, respectively.

Carrying out the usual anti-commutation algebra we arrive at the integral

$$\int d^3k\, d^3k'\, \delta(\vec{k}-\vec{p}_1)\delta(\vec{k}'-\vec{p}_2)\, e^{i(E+E')t-i(\vec{k}+\vec{k}')\vec{x}}\,. \tag{10}$$

The integral can be trivially taken and substituted into the integral (8) over $d^3x dt$. Integration over d^3x gives $\delta(\vec{p}_1+\vec{p}_2)$ and we are left with the Fourier transform [a]

$$A(\vec{p}_1,\vec{p}_2) \sim g^2 \delta^{(3)}(\vec{p}_1+\vec{p}_2)\int dt\, \phi(t)\, e^{i(E_1+E_2)t}\,. \tag{11}$$

The probability of particle production is proportional to $|A(\vec{p}_1,\vec{p}_2)|^2$ and contains the square of momentum delta-function. The latter is treated in the standard way,

$$[\delta(\vec{p}_1+\vec{p}_2)]^2 = 2\pi V \delta(\vec{p}_1+\vec{p}_2)\,, \tag{12}$$

where V is the total space volume. The origin of the volume factor is evident: since the external field is space-point independent, so is the probability of production per unit volume and the total probability is proportional to the total volume.

Similar situation is realized for the time dependence in the case of periodic external fields, if one neglects back reaction of the produced particles on the field evolution and on the probability of production. The former can be taken into account by a (slow) decrease of the field amplitude $\phi_0(t)$, while the latter is determined by the statistics of the produced particles: the probability of boson production is proportional to the phase space density of already produced bosons, $(1+f_k)$, while the probability of fermion production is inhibited by the factor $(1-f_k)$. This back reaction effect is absent for Boltzmann statistics, which we will mostly assume in what follows. Thus, for a periodic external field one would expect that the probability of production is proportional to the total time interval, during which the external field was operating. In the idealistic case of $\phi \sim \exp(i\omega t)$, its Fourier transform gives $\delta(2E-\omega)$, and the square of the latter is, as above, $t_{tot}\delta(2E-\omega)$. The second factor ensures energy conservation and is infinitely large for $E=\omega/2$. It means that the phase space density of the produced particles becomes very large after period of time when the energy conservation is approximately established. One can check that this time is much shorter than $1/\Gamma$ (where Γ is the perturbative decay rate); still, the time of transition of energy from the inflaton field to

[a] For details and more rigorous consideration in terms of Bogolyubov coefficients see, e.g., Appendix A in Ref. 25.

the produced fermions is given by $1/\Gamma$. This fact is commonly agreed upon in the case of perturbative production. The statement in the literature that in non-perturbative regime fermion production could be very strong is possibly related to this trivial rise of the occupation numbers and does not mean that fermion production can compete with production of bosons (see below).

In the case when external field operates during a finite period of time, starting e.g. from $t = 0$, or if one is interested in the number of produced particles at the running moment t, the integral in expression (11) should be taken in the limits $(0, t)$ and for the particular case of $\phi = \phi_0 \cos \omega t$ one obtains:

$$\begin{aligned} I(t; E, \omega) &\equiv \int_0^t dt\, e^{2iEt} \cos \omega t \\ &= e^{i(E-\omega/2)t} \left[\frac{\sin(E-\omega/2)t}{2E-\omega} + e^{i\omega t} \frac{\sin(E+\omega/2)t}{2E+\omega} \right] . \end{aligned} \tag{13}$$

For E close to $\omega/2$ the first term dominates and the number of produced fermions rises as t^2 until $t \sim 1/|2E-\omega|$. At larger times it oscillates. The same phenomena was found in non-perturbative calculations. Indeed, the phase space number density of the produced particles (we use this term interchangeably with the "occupation number") is given by

$$f_p = g^2 \phi_0^2 \mid I(t; E, \omega) \mid^2 . \tag{14}$$

As we have argued above, usually one has $\mid I(t; E, \omega) \mid^2 = 2\pi t\, \delta(2E - \omega)$. In this case the number density of the produced particles as a function of time is given by

$$n(t) = \int \frac{d^3 p}{(2\pi)^3} f_p = \frac{g^2 \omega}{8\pi} \phi_0^2 \omega t = \Gamma n_\phi t\,, \tag{15}$$

where $n_\phi = \phi_0^2 \omega$ is the number density of ϕ-bosons and Γ is their decay width given by Eq. (7).

A detailed explanation of the discussed phenomena can be found in textbooks on quantum mechanics in the section where perturbation theory for time dependent potential is presented, see e.g. Ref. 26.

Returning to the occupation number (14) we see that for $(\omega - 2E)t < 1$ it evolves as $f_p \approx g^2 \phi_0^2 t^2$ and reaches unity at $t = t_1 = 1/g\phi_0$. This is much earlier than $t_d = 1/\Gamma$ which is the characteristic decay time of $\phi(t)$:

$$\frac{t_d}{t_1} = \frac{8\pi}{g} \frac{\phi_0}{\omega} . \tag{16}$$

Taken formally, this ratio may reach the value $10^8 - 10^9$. This is an explanation of statement that fermions could be very quickly produced by inflaton. On the other hand, although some fermionic bands (approximately satisfying energy conservation) might be quickly populated, the total transfer of energy from the inflaton to the produced particles is determined by the total decay rate and is much slower. Roughly speaking, $f_p = 1$ corresponds to production of only one pair of fermions and, of course, the energy of this pair is negligible in comparison with the total energy accumulated in the classical field $\phi(t)$.

Perturbation theory is not applicable if the effective mass of fermions $m_{eff}=(m_0+g\phi_0)$ is larger than the frequency of the oscillations of the scalar field. For example, the probability of pair production by two-quanta process, in which the energy of each produced fermion is equal to ω, is related to that of one-quantum process, $E = \omega/2$, as $W_2/W_1 \sim (g\phi_0/\omega)^2$. It is still possible that $\phi_0/g\omega \gg 1$, while $g\phi_0/\omega < 1$, so that perturbation theory is reliable and the relation $t_d/t_1 \gg 1$ still holds. However in many practically interesting cases $g\phi_0/\omega > 1$ and in this range of parameters the result obtained above can serve only for the purpose of illustration and for more precise statements we have to go beyond perturbation theory. This will be done in the following section by the imaginary time method.[27,28,5] (For recent applications of this method and a more complete list of references see Refs. 29.) It is qualitatively clear that non-perturbative effects could only diminish the rate of particle production because the non-perturbative calculations take into account non-vanishing and large value of the effective mass of the produced particles and this leads to a smaller rate of the production in comparison with the case when the interaction is taken in the form $g\phi\bar{\psi}\psi$ but its contribution into fermion effective mass is neglected. As we see below, the suppression of the production rate in nonperturbative regime[6] in comparison with perturbation theory is given by the factor $(\omega/g\phi)^{1/2}$ in qualitative agreement with these simple arguments.

Effects of quantum statistics were neglected above, and thus the results obtained are valid only if $f_p < 1$. The corresponding corrections can be approximately introduced by multiplication of the r.h.s. of Eq. (14) by the factor $(1 \pm f_p)$ and correspondingly, $f_p^{(f,b)} = g^2\phi_0^2|I|^2/(1 \pm g^2\phi_0^2|I|^2)$, where the "±" signs refer to fermions and bosons respectively. One sees that the production of fermions effectively stops (as one should expect) when $f_p^{(f)} \sim 1$, while production of bosons tends to infinity. Presumably a more accurate treatment would not allow bosons to reach infinitely large density in a finite time but the message is clear, the production of bosons becomes explosive in perturbation theory with characteristic time of the order of $t_1 = 1/(g\phi_0)$ and all the energy of the inflaton would go into that of the produced bosons during approximately this time. There are several effects that can weaken this conclusion. One is a possible inapplicability of perturbation theory for a large $g\phi_0/\omega$. This effect qualitatively acts in the same way as in fermionic case discussed above. Still, even if the $g\phi_0/\omega \gg 1$ the effect of explosive production of bosons would survive due to parametric resonance in equation of motions for the produced modes.[6,22,24] Another two effects that could diminish the production are the cosmological red-shift of momenta of the produced particles and their scattering on other particles in the background. Both would push the produced particles away from the resonance band and could significantly slow down the production in the case of narrow resonance,[6] while in the case of wide resonance the effect survives.[22,24]

On the other hand, both red-shift and scattering of the produced fermions back react on their production in exactly opposite (to bosons) way. These phenomena "cleans" the occupied zone and allows for production of more fermions.

4 Quasiclassical limit: imaginary time method

4.1 Small mass case

Usually, non-perturbative calculations are not simple but in the case that we are considering there is a fortunate circumstance that in the anti-perturbative limit quasiclassical approximation works pretty well. The latter can be efficiently treated by the imaginary time method.[27,28,5] Below we will essentially repeat Ref. 6 correcting some typos and algebraic errors, though the basic results of the paper remain intact.

The coupling of $\phi(t)$ to the produced particles is equivalent to prescription of the time dependent mass to the latter, $m(t) = m_0 + g\phi(t)$. The classical Lagrange function for a relativistic particle with such a mass has the form

$$L_{cl} = -m(t)\left(1 - \vec{V}^2\right)^{1/2}, \tag{17}$$

where $\vec{V}$ is the particle velocity. The corresponding Hamiltonian is

$$\mathcal{H} = \left[p^2 + m^2(t)\right]^{1/2} \equiv \Omega(t)\,. \tag{18}$$

The quantization of this system can be achieved by the path integral method. The Green's function of the quantum particle has the form (see e.g. Ref. 4):

$$G(\vec{x}_f, t_f; \vec{x}_i, t_i) = \int D\vec{p}\, D\vec{x}\, \exp\left[i\int_{t_i}^{t_f} dt\left(\vec{p}\dot{\vec{x}} - \mathcal{H}\right)\right]. \tag{19}$$

This functional integral can be easily taken:

$$G(\vec{x}_f, t_f; \vec{x}_i, t_i) = \int \frac{d^3p}{(2\pi)^3}\, \exp\left[i\vec{p}\,(\vec{x}_f - \vec{x}_i) - i\int_{t_i}^{t_f} dt\, \Omega(t)\right]. \tag{20}$$

According to the general rules of quantum mechanics the amplitude of the transition from the state given by the initial wave function Ψ_i into that given by Ψ_f is equal to

$$A(\vec{p}_1, \vec{p}_2) = \int d^3x_i \, d^3x_f \;\; \Psi_f^*(x_f) \, G(\vec{x}_f, t_f; \vec{x}_i, t_i) \Psi_i(x_i) \,, \tag{21}$$

where plane waves are usually substituted for $\Psi_{i,f}$.

To obtain the amplitude of creation of a pair of particles the contour of integration over time should be shifted into complex t-plane in such a way that it goes around the branching point of the energy Ω in the direction of changing the sign of energy from negative to positive. This corresponds to transition from the lower continuum of the Dirac sea to the upper one, i.e. to pair creation. Thus, we find

$$A(\vec{p}_1, \vec{p}_2) = (2\pi)^3 \, \delta \, (\vec{p}_1 + \vec{p}_2) \exp \left[-\, i \int_{C(t_i, t_f)} dt \; \Omega(t) \right], \tag{22}$$

where the contour $C(t_i, t_f)$ starts at $t = t_i$, ends at $t = t_f$, and turns around the branching point of Ω in the way specified above.

The position of the branching points $t_b = t' + it''$ can be found from

$$p^2 + (m_0 + g\phi_0 \cos \omega t)^2 = 0 \,. \tag{23}$$

Correspondingly,

$$m_0 + g\phi_0 \, (\cos \tau' \cosh \tau'' - i \sin \tau' \sinh \tau'') = \pm ip \,, \tag{24}$$

where $\tau = \omega t$. In what follows we assume that $m_0 = 0$ and it will grossly simplify technical details. In this limit $\tau' = \pi/2 + n\pi$ and $\sinh \tau'' = \pm (p/g\phi_0)$.

The integral along the cut $\tau = \tau' + i\eta$ is real and, according to our prescription, negative. It gives exponential suppression factor for the production probability, $W \sim \exp(-2Q)$, with

$$Q = \frac{2}{\omega} \int_0^{\tau''} d\eta \left(p^2 - g^2 \phi_0^2 \, \sinh^2 \eta \right)^{1/2} . \tag{25}$$

This integral can be expressed through complete elliptic functions as[30]

$$Q = \frac{2\sqrt{p^2 + m_1^2}}{\omega} \left[K\,(\beta) - E(\beta) \right], \tag{26}$$

where

$$m_1 = g\phi_0 \,, \quad \text{and} \quad \beta = p / \sqrt{p^2 + m_1^2} \,. \tag{27}$$

For small β these functions can be expanded as $K(\beta) \approx (\pi/2)(1+\beta^2/4)$ and $E(\beta) \approx (\pi/2)(1-\beta^2/4)$, so that $Q \approx (\pi/2)(p^2/\omega\, m_1)$.

The total production amplitude is equal to the sum of expressions (22) with all the contours encircling the proper branch points between t_i and t_f. Since the integrals along imaginary direction $id\eta$ are all real and have the same value for all branch points, their contribution to the amplitude gives the common factor $\exp(-Q)$. The integrals over real time axis corresponding to different contours C around neighboring branch points differ by the phase factor $A_{n+2}/A_n = \exp(2i\alpha)$, because the energy changes sign after the integration contour turns around branch points. The absence of the contribution from the nearest cut is related to the particle statistics and is discussed, e.g., in Refs. 28 and 5. The phase α is given by:

$$\alpha = \int_0^{2\pi} dt\, \sqrt{p^2 + m_1^2 \cos^2 \omega t} = \frac{4\sqrt{p^2+m_1^2}}{\omega}\, E\left(\sqrt{1-\beta^2}\right). \tag{28}$$

All this is true if the free fermion mass is vanishing, $m_0 = 0$, otherwise equations become significantly more complicated. In the limit of small β we find[30]

$$E\left(\sqrt{1-\beta^2}\right) \approx 1 + \frac{\beta^2}{2}\left(\ln\frac{4}{\beta} - \frac{1}{2}\right), \tag{29}$$

while for β close to 1 the necessary expressions are presented after Eq. (27) with the interchange $\beta^2 \leftrightarrow (1-\beta^2)$.

Summing over all branch points we obtain:

$$A(\vec{p}_1, \vec{p}_2) = (2\pi)^3\, \delta\left(\vec{p}_1 + \vec{p}_2\right) \exp\left(-Q + i\alpha\right) \frac{\sin(N\alpha) - 1}{\sin\alpha - 1}, \tag{30}$$

where N is the total number of branch points included in the amplitude; it is approximately equal to the total time in units $1/\omega$ during which the particles are produced, $N = \text{Integer}[(t_f - t_i)/\omega]$. The last factor reminds that coming from the integration over time in perturbation theory discussed in Sec. 3 and in fact its physical nature is the same. For very large N, formally for $N \to \infty$, it tends to

$$\frac{\sin(N\alpha)}{\sin\alpha} \to \pi \sum_j \delta\left(\alpha - \pi j\right). \tag{31}$$

These delta-functions impose energy conservation for the production of pair of particles by j quanta of the field ϕ. Note that in contrast to the lowest order perturbation theory, when only a single quanta production is taken into account, the expression (30) includes production of a pair by many quanta of the field ϕ. For example, in the limit of high momenta of the produced particles these delta-functions are reduced to $\delta(2p - j\omega)$, the same as in perturbation theory for j-quanta production.

Treating again, as in Sec. 3, the square of delta-function as a product of the single delta-function and $\delta(0) = \pi N$ with N expressed through the total time t, during which the particles have been produced, as the integer part of $t\omega$, we find the following expression for the rate of production per unit time and unit volume,[6]

$$\dot{n} = \pi\,\omega \sum_j \int \frac{d^3p}{(2\pi)^3}\,\exp(-2Q)\,\delta\,(\alpha - \pi j)\,. \tag{32}$$

In the limit of $m_1 \gg \omega$ one obtains

$$Q \;\approx\; \frac{\pi}{2}\,\frac{p^2}{\omega\, m_1}\;, \tag{33}$$

$$\alpha \;\approx\; \frac{4m_1}{\omega}\left[1 + \frac{p^2}{2m_1^2}\left(\ln\frac{4m_1}{p} + 1\right)\right], \tag{34}$$

and hence,

$$\dot{n} = \frac{1}{2\pi}\sum_{j_m}\left\{\exp\left[-\frac{\pi^2\,(j - (4m_1/\pi\omega))}{\ln(4m_1/p_j) + 1}\right]\frac{\omega^2 m_1 p_j}{\ln(4m_1/p_j) + 1/2}\right\}. \tag{35}$$

Here summation starts from the minimum integer value $j_m \geq (4m_1/\pi\omega)$ and p_j is determined from the equation $\alpha = j\pi$, i.e.

$$p_j^2 \approx \frac{(\pi m_1\omega/2)\,(j - 4m_1/\pi\omega)}{\ln(4m_1/p_j) + 1}\;. \tag{36}$$

A rough estimate gives $\dot{n} \sim \omega^{5/2} m_1^{3/2}$. Correspondingly, the characteristic rate of the inflaton decay in the quasiclassical approximation is given by

$$\Gamma_q = \dot{n}/n_\phi = \dot{n}/(\omega\phi_0^2) \sim \Gamma\,(\omega/m_1)^{1/2}\,, \tag{37}$$

where Γ is the decay rate in perturbation theory (7). One sees that in the quasiclassical limit the decay rate is suppressed in comparison with the formal result of perturbation theory as a square root of the ratio of the oscillation frequency to the amplitude of the scalar field. This suppression can be understood as follows.[6] Most of the time the instant value of the field $\phi(t)$ and the effective mass of the fermions, $m_{eff} = g\phi_0 \cos\omega t$ are large in comparison with the oscillation frequency. As it is well known (see also Sec. 4.2 below), the probability of particle production in this case is exponentially suppressed. However, when $\cos\omega t$ is very close to zero the effective mass of the produced particles is smaller than ω and they are essentially produced at this short time moments. This results in a much milder suppression of the production, not exponential but only as $(\omega/g\phi_0)^{1/2}$.

In case of finite and not too big N we will see that, according to the calculations of Ref. 6 presented above, the occupation number f_p would reach unity in a much shorter time than $1/\Gamma_q$. This result was rediscovered later in Refs. 31 and 32 by numerical calculations and reconfirmed by analytical methods in Ref. 33. However, as it has been already argued, this does not mean that non-perturbative production of fermions is strong, it is always weaker than the perturbative one.

The calculations presented above do not include the effects of quantum statistics, so strictly speaking, they are valid for "boltzons". Thus, they present an upper bound for the production of fermions. In the fermionic case, the production would stop when the occupation number, f_p, approaches unity, while production of "boltzons" would go unabated. However, if the particles from the occupied Fermi band are quickly removed by scattering or red-shift (as we discussed above) the production of fermions would go essentially with the same rate as production of "boltzons".

For a finite number of oscillations N the occupation number of the produced particles is equal to (see Eq. (30))

$$f_p(N) = \exp(-2Q) \left(\frac{\sin(N\alpha) - 1}{\sin\alpha - 1} \right)^2 . \tag{38}$$

The last factor is rather similar to that in Eq. (13). This is an oscillating function of N. For $\alpha = \pi(1-\epsilon)$ with a small ϵ it rises roughly as N^2 during $N = 1/(2\epsilon)$ oscillations. The occupation number increases with time discontinuously as a series of discrete jumps as time t/ω reaches integer values. During this stage f_p may quickly rise with the speed much faster than the rate Γ_q, see Eq. (37), in complete analogy with the perturbative case considered above in Sec. 3. However, as we have already stressed, this does not mean that the production of fermions goes faster than in perturbation theory.

After this period of increase, f_p starts to go down and approaches zero at $N_0 \approx 1/\epsilon$. This oscillating behavior of the number of produced particles was noticed long ago in the problem of e^+e^--pair creation by periodic electric field (for the list of references see, e.g., the book by Grib *et al* in Ref. 12). Thus, it looks as though particles are produced by the field and after a while they all are absorbed back. This behavior is difficult to digest. Note that it is absent if time is very large, tending to infinity, as it was discussed above. In this case the energy conservation is strictly imposed by the delta-function, $\alpha = \pi n$ (where n is an integer), or in other words, $\epsilon = 0$ and $N_0 \to \infty$.

Possibly this mysterious phenomenon of re-absorption of the produced particles is related to the fact that during finite time the external field $\phi(t)$ does not disappear and the particle vacuum is not well defined over this time dependent background. To resolve the ambiguity one may calculate the transition of energy from the time-varying field $\phi(t)$ into other quantum states which are not necessarily determined in terms of particles. Energy density, in contrast to the particle number density, can be unambiguously defined in terms of local fields operators and does not suffer from any ambiguity related to the non-local character of the latter. The energy density of the quantum field ψ, defined as the expectation value of the time-time component of its energy-momentum operator, may also exhibit the oscillating behavior described above but the correct interpretation is possibly not production of ψ-particles but some excitation ("classical"?) of the (fermion) field ψ coupled to $\phi(t)$.

4.2 Large mass case

Let us now consider the case where the fermion mass m_0 is large in comparison with the oscillation frequency ω and with the amplitude of the oscillations, $m_0 \gg g\phi_0$, so that the total effective fermion mass $m_{tot} = m_0 + g\phi_0 \cos\omega t$ never vanishes and always large. The calculations for this case have been only done in Ref. 6 and we will reproduce them here. To be more precise, we will reproduce only imaginary time part, while in Ref. 6 the method of Bogolyubov coefficients was used as well.

Following this paper we will consider production of bosons. It will be technically simpler allowing to make all calculations analytically. Qualitatively the same results should be valid also for fermions, because for a large m_0 the production is weak and the occupation numbers remain small. We assume that the effective mass has the form

$$m^2(t) = m_0^2 + g^2\phi_0^2 \cos^2\omega t\,. \tag{39}$$

This case is realized if the interaction of the inflaton field with the produced particles (χ-bosons) has the form $g^2|\chi^2|\phi^2$. The probability of production can be found from the expressions of the previous subsection by the substitution $p^2 \to p^2 + m_0^2$. In particular, the exponential damping factor is given, instead of (26), by

$$Q' = \frac{2\sqrt{p^2+m_0^2+m_1^2}}{\omega}\left[K\left(\beta'\right) - E(\beta')\right], \tag{40}$$

where

$$(\beta')^2 \equiv 1 - u^2 = 1 - \frac{m_1^2}{m_0^2+m_1^2+p^2} \tag{41}$$

and the complete elliptic integrals in the case of small k are expanded as[30]

$$\begin{aligned} K(\beta') &\approx \ln\frac{4}{u} + \frac{u^2}{4}\left(\ln\frac{4}{u} - 1\right) \\ E(\beta') &\approx 1 + \frac{u^2}{2}\left(\ln\frac{4}{u} - \frac{1}{2}\right). \end{aligned} \tag{42}$$

The phase difference over the period of oscillations is now given by

$$\begin{aligned} \alpha' &= \frac{4\sqrt{p^2+m_0^2+m_1^2}}{\omega} E\left(\sqrt{\frac{m_1^2}{m_1^2+m_0^2+p^2}}\right) \\ &\approx \frac{2\pi\sqrt{p^2+m_0^2+m_1^2}}{\omega}\left(1 + \frac{m_1^2}{4(m_1^2+m_0^2+p^2)}\right). \end{aligned} \tag{43}$$

We can repeat the same calculations as in the previous subsection to find the occupation number and the number density of the produced particles. The production probability is now exponentially suppressed, as

$$\exp\left(-\frac{2\sqrt{m_0^2+m_1^2}}{\omega}\ln\left[\frac{16(m_0^2+m_1^2)}{m_1^2}\right]\right).$$

For a sufficiently large ratio m_0/ω the production would be very weak, all occupation numbers would be small in comparison with unity and bosons and fermions would be equally poorly produced.

5 Back reaction and cosmological expansion effects

Now we briefly comment on applicability of the results discussed above to realistic case of universe (re)heating after inflation. We have neglected universe expansion and damping of the field ϕ due to energy transfer to the produced particles. The effect of expansion can be easily taken into account in conformal coordinates where the metric takes the form (1) with space point independent cosmological scale factor $a(\tau)$. Under transformation to conformal coordinates and simultaneous redefinition of the gravitational, scalar, and fermionic fields as $g_{\mu\nu} \to a^2 g_{\mu\nu}$, $\phi \to \phi/a$, and $\psi \to \psi/a^{3/2}$, respectively, the mode equation for the scalar field takes the form

$$\phi_k'' + (k^2 + m^2 a^2 - a''/a)\,\phi_k = 0\,, \tag{44}$$

where the derivatives are taken with respect to conformal time and k is comoving momentum. The presence of the term a''/a demonstrates breaking of conformal invariance even for massless scalar field, as it has been already mentioned in Sec. 2. All masses enter equation of motion in the combination ma, so mass terms explicitly break conformal invariance. The interactions of the types $g\phi\bar{\psi}\psi$, $\lambda\phi^4$, and $f\phi^2\chi^*\chi$ are invariant with respect to the transformation of the fields specified above (note that the presence of the $\sqrt{det[g_{\mu\nu}]}$ in the action integral gives the necessary factor a^4 to ensure this invariance).

The expressions for the scale factors through conformal time in three most interesting cosmologies are the following:

$$\begin{aligned} &a \sim e^{Ht} = -1/H\tau && \text{DeSitter universe, inflation,} \\ &a \sim t^{1/2} \sim \tau && \text{radiation dominance,} \\ &a \sim t^{2/3} \sim \tau^2 && \text{matter dominance.} \end{aligned} \tag{45}$$

In particular, in the radiation dominated universe with conformally invariant interactions, scalar field is conformally invariant but this is not true for other expansion regimes. Correspondingly, particles production by massless scalar field with the self-potential $\lambda\phi^4$ can be reduced to the flat space case discussed in the previous section. The difference between the potentials of ϕ in these two cases, $\omega^2\phi^2$ and $\lambda\phi^4$, is not essential and the obtained above results can be easily translated to the $\lambda\phi^4/4$ potential. Indeed, the equation of motion of spatially homogeneous field ϕ in flat space-time (in conformal coordinates) is

$$\phi'' + \lambda\phi^3 = 0\,. \tag{46}$$

This equation is solved in terms of Jacobi elliptic functions:[30]

$$\begin{aligned} \phi(\tau) &= \phi_0\, \mathrm{cn}\left(\sqrt{2\lambda}\phi_0\tau; \sqrt{2}\right) \\ &= \frac{\sqrt{2}\pi}{\kappa} \sum_{n=1} \frac{\cos\left[(n-1/2)\pi\sqrt{2\lambda}\phi_0\tau/\kappa\right]}{\cosh\left[(n-1/2)\pi\right]}\,, \end{aligned} \tag{47}$$

where $\kappa = \Gamma^2(1/4)/4\sqrt{\pi}$. The expansion is well approximated by the first term and particle production rate can be estimated using results of the previous section. Significant deviations from those results can be expected only in the case of heavy particle production when higher frequency terms in expansion (47) may compete with the exponentially suppressed contribution coming from lower terms (see Eq. (40)).

It should be repeated, however, that these results are true only for radiation dominated regime of expansion. For other cosmologies the term a''/a in Eq. (44) is non-vanishing and must be taken into account.

Another effect, in addition to expansion, that results in a decrease of the amplitude of the field $\phi(t)$, is back reaction of the particle production. Energy that is transferred to the produced particles is taken from the field ϕ so the energy density of the latter should become smaller. For harmonic oscillations (in the case of the potential $\omega^2\phi^2$) only the amplitude of the field diminishes, while frequency remains the same. For quartic potential both the frequency and the amplitude of oscillations go down, as one can see from Eq. (47) with $\phi_0(t)$.

In the case of quickly oscillating field the effect can be easily estimated in adiabatic approximation. One has to solve the equation for energy balance in expanding background:

$$\dot\rho = -3H\,(\rho + P)\,, \tag{48}$$

where ρ and P are respectively energy and pressure densities of the field ϕ and the produced particles. For the former the solution of the standard equation of motion without interactions should be substituted with the effect of production included in a slow decrease of the amplitude ϕ_0.

More accurate consideration demands using equation of motion modified by the production process. Usually this is described by the introduction of the "production friction term" into equations of motion, in addition to Hubble friction,

$$\ddot\phi + 3H\dot\phi + U'(\phi) = -\Gamma\dot\phi\,, \tag{49}$$

where $U(\phi)$ is the potential of ϕ and its derivative $U'(\phi)$ is taken with respect to ϕ. This anzats gives reasonable results only for harmonic potential but in all other cases this approximation is not satisfactory. A better approximation has been derived in Refs. 34 and 35. One starts with exact quantum operator equation of motion for the field ϕ and some other fields χ that are coupled to ϕ. The production of the latter by oscillations of ϕ results in a damping term in the equation of motion for ϕ. As an example let us consider a simple case of scalar χ with trilinear coupling $f\phi\chi^2$. The corresponding equations of motion are (expansion is neglected for simplicity)

$$\ddot{\phi} - \Delta\phi + V^{'}(\phi) = f\chi^2, \tag{50}$$

$$\partial^2\chi + m_\chi^2\,\chi = 2f\phi\chi\,. \tag{51}$$

The next step is to make quantum averaging of these equations in the presence of classical field $\phi_c(t)$ (in what follows we omit sub-c and neglect the mass of χ). This can be easily done in one-loop approximation,[b] and one comes to the equation that contains only the field ϕ and accounts for the back reaction from the production of the quanta of χ:

$$\ddot{\phi} + V'(\phi) = \frac{f^2}{4\pi^2}\int_0^{t-t_{in}} \frac{d\tau}{\tau}\,\phi(t-\tau)\ , \tag{52}$$

where t_{in} is an initial time, when the particle production was switched on (it is assumed that $t > t_{in}$). The term in the r.h.s. that describes the influence of the particle production is non-local in time as one should have expected because the impact of the produced particle on the evolution of ϕ depends upon all the previous history. To use this equation for realistic calculations one has to define a proper renormalization procedure. It is described in detail in Ref. 35. The coupling to fermions as well as quartic coupling $\lambda'\phi^2\chi^2$ are also considered in that paper. Similar one-loop approach was used in Ref. 31 but no self-contained equation for ϕ was derived there.

Both effects, cosmological expansion and damping of ϕ due to particle production, can be easily incorporated into imaginary time method. This is especially simple in the case of fast oscillations and slow decrease of the amplitude of ϕ. In this case the results obtained above practically do not change. One should only substitute $\phi_0(t)$ there and determine the law of the evolution of the latter from the energy balance equation (48) or, more accurately, from Eq. (52).

One more phenomenon deserves a comment here. As we have already mentioned, production of bosons may be strongly amplified due to the presence of the earlier produced bosons in the same final state. In classical language this effect is described by the parametric resonance in the equation of motion of the produced particles, while in quantum language it is the so called stimulated emission well known in laser physics. When the amplitude of the driving field

[b] Some subtleties related to renormalization of mass and coupling constants are discussed in Ref. 35.

ϕ drops below a certain value, the resonance would not be excited and the rest of ϕ would decay slowly. If the mass of ϕ is non-zero, this field behaves as non-relativistic matter and its cosmological energy density drops as $1/a^3$. On the other hand, the produced particles are mostly relativistic with energy density decreasing as $1/a^4$. Thus, for a sufficiently slow decay rate of ϕ, the latter may dominate the cosmological energy density once again, when previously produced particles are red-shifted away. This would result in a low second reheating temperature, much lower than in parametric resonance scenario. On the other hand, the phenomenon of stimulated emission persists in perturbation theory even with a very small amplitude of ϕ. Possibly even in this limit the production is not very fast as well, because the width of the band is quite narrow and the produced bosons are quickly pushed away from the band due to cosmological red-shift and collisions. More detailed consideration is desirable here.

6 Conclusion

It is demonstrated that imaginary time method very well describes particle production by scalar field. It is very simple technically and permits to obtain physically transparent results. The calculations were done here for a particular case of periodic or quasiperiodic oscillations of the field but, as the experience with production of e^+e^--pairs by electric field shows (for a review see, e.g., the third paper in Ref. 5), the method also works well in the opposite case of short pulse fields. The method is applicable in the quasiclassical limit. In the opposite case perturbation theory is applicable, and hence, one can obtain simple and accurate (semi)analytical estimates practically in all parameter range.

The results of calculations in the quasiclassical limit are in a good agreement with subsequent numerical ones.[31,32] An important difference between the latter papers and the initial one[6] lays in the interpretation of the results. According to all these papers the occupation numbers of the produced particles quickly approaches unity but, in contrast to Refs. 31 and 32, it is argued in Ref. 6 that the total production rate is nevertheless suppressed in comparison to perturbation theory and the production of fermions by the inflaton with Yukawa coupling to fermions is always weak. This conclusion is verified above. As shown in this paper, the occupation numbers may quickly reach unity both in perturbation theory and in non-perturbative case. Still the production rate, even for particles obeying Boltzmann statistics, is very weak to ensure fast (pre,re)heating. In the case of fermion production the rate is evidently much weaker because the production must stop when the occupation number reaches unity and to continue the process the produced fermions should be eliminated from the band. As it is argued in Sec. 4.1, the non-perturbative effects can only diminish the production rate.

The bosonic case is opposite: the more bosons there are in the final state, the faster is production. Thus, even in perturbation regime the boson production can be strongly amplified because their occupation number may reach unity in much shorter time than $1/\Gamma$ and the energy may be transferred from the inflaton to the produced bosons much faster than it is given by the original perturbative estimates,[21] where the effect of stimulated emission is not taken into account. Of course, to realize this regime the band should not be destroyed by expansion and scattering, as argued in Ref. 6.

To summarize, we have shown that perturbation theory gives a good estimate of production of light fermions and bosons if Fermi exclusion principle or stimulated emission, respectively, are taken into account. The formally calculated production rate in perturbation theory is always larger than the non-perturbative one, at least in the simple cases that we have considered. So, the results of perturbation theory may be used as upper bounds for production rates. Moreover, perturbation theory helps to understand physical meaning of the obtained results and to interpret them correctly.

In many realistic cases (e.g. for large $g\phi_0$ or m_0) perturbation theory is not applicable, and to calculate the real production rate (not just an upper bound) one has to carry out more involved non-perturbative calculations. In quasiclassical (anti-perturbative) limit the imaginary time method permits to obtain accurate and simple results and to avoid complicated numerical procedure.

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