DECOHERENCE AS A FUNDAMENTAL PHENOMENON IN QUANTUM DYNAMICS

MICHAEL B. MENSKY
P.N. Lebedev Physical Institute, 117924 Moscow, Russia

The phenomenon of decoherence of a quantum system caused by the entanglement of the system with its environment is discussed from different points of view, particularly in the framework of quantum theory of measurements. The selective presentation of decoherence (taking into account the state of the environment) by restricted path integrals or by effective Schrödinger equation is shown to follow from the first principles or from models. Fundamental character of this phenomenon is demonstrated, particularly the role played in it by information is underlined. It is argued that quantum mechanics becomes logically closed and contains no paradoxes if it is formulated as a theory of open systems with decoherence taken into account. If one insist on considering a completely closed system (the whole Universe), the observer's consciousness has to be included in the theory explicitly. Such a theory is not motivated by physics, but may be interesting as a metaphysical theory clarifying the concept of consciousness.

Contents

1 Introduction

It is honor for me to contribute to the volume in memory of my friend Michael Marinov, particularly because this gives a good opportunity to recall one of his early innovatory works which was not properly understood at the moment of its publication. I mean the work started in Refs. 1 and 2 and continued later in Ref. 3. In these papers Misha Marinov considered a sort of nonunitary evolution of a quantum system converting a pure state into a mixed state. Such a behavior appears in open quantum systems as a consequence of the phenomenon which became known later as decoherence. Decoherence was rediscovered several times in different approaches (see for example the reviews[4–6] and references therein). One of these approaches, based on restricted path integrals, will be discussed in detail in the present paper.

Importance of the phenomenon of decoherence is now generally recognized, but the first two papers by M. Marinov[1,2] evoked only perplexity. Unitarity seemed to be necessary feature of any quantum evolution. Eventually it became clear that the evolution of an open system may be non-unitary and must be thought of as a specific type of evolution. This idea was formulated by various authors in various forms, often independently from each other. In many cases master equations, i.e. equations for density matrices, were used just as in the papers of M. Marinov. The most general form of such an equation was suggested by G. Lindblad.[7] However other approaches were also proposed for presentation of decoherence, among them: consistent histories,[8–12] quantum trajectories,[13] stochastic equations,[14–16] and restricted path integrals (called also quantum corridors).[17–20,5,6] The last two approaches provide the so-called selective description of the process in which a pure initial state remains pure despite of the decoherence.

I did not keep up with the literature on this subject until I discovered that the formalism of restricted path integrals developed by me in order to describe continuous quantum measurements, represent in fact the process of gradual decoherence. It became clear later on that this approach to decoherence reveals very interesting conceptual features of the phenomenon. Below I shall discuss this in some detail.

The plan of the paper is following. In Sec. 2 it is demonstrated, with the help of the simplest model, how entanglement of a system with its environment leads to the systems's decoherence. In Sec. 3 the idea is presented of the restricted-path-integral approach to continuous quantum measurements or, what is the same, to gradual decoherence. In Secs. 4, 5 monitoring an observable is considered as a special but very important example of continuous measurements. Fundamental aspects of decoherence are discussed in Sec. 6. In

Sec. 7 expansion of quantum mechanics beyond the theory of decoherence is shortly commented. It is argued that such an expansion is not in fact physically motivated, but is interesting, particularly because it requires the observer's consciousness to be included in the theory explicitly. A short resume of the paper is given in Sec. 8.

2 Decoherence by entanglement with an environment

A pure state of a quantum system may be described by the wave function or state vector $|\psi\rangle$. Instead, it may be presented also by the density matrix $\rho = |\psi\rangle\langle\psi|$. If the state $|\psi\rangle$ is a superposition of some other states $|a\rangle$ and $|b\rangle$,

$$|\psi\rangle = \alpha|a\rangle + \beta|b\rangle \, , \tag{1}$$

the density matrix takes the form

$$\rho = |\alpha|^2|a\rangle\langle a| + \alpha\beta^*|a\rangle\langle b| + \beta\alpha^*|b\rangle\langle a| + |\beta|^2|b\rangle\langle b| = \begin{bmatrix} |\alpha|^2 & \alpha\beta^* \\ \beta\alpha^* & |\beta|^2 \end{bmatrix} . \tag{2}$$

A mixed state may be presented only by a density matrix. The mixture of the states $|a\rangle$ and $|b\rangle$ (with the weights $|\alpha|^2$ and $|\beta|^2$) is presented by the density matrix

$$\rho = |\alpha|^2|a\rangle\langle a| + |\beta|^2|b\rangle\langle b| = \begin{bmatrix} |\alpha|^2 & 0 \\ 0 & |\beta|^2 \end{bmatrix} , \tag{3}$$

differing from (2) by the off-diagonal matrix elements being zero.

The relative phase of the complex coefficients α and β in the superposition (1) or (2) is essential. In order to underline this fact, the state (1) or (2) is sometimes called *coherent superposition*. On the contrary, no phase at all appears in the definition of mixture (3). The process converting the pure state (2) into the mixed state (3) may be described as nullifying the off-diagonal elements of the density matrix. As a result, the density matrix becomes diagonal in the given basis. This process is called *decoherence*. We consider here only the simplest example of decoherence, but it is enough to demonstrate the idea.

Decoherence may occur in the course of interaction of the system $\mathcal{S}$ with its environment $\mathcal{E}$. This means that the interaction results in diagonalizing the systems's density matrix in a certain basis. Of course, the concrete basis in which the density matrix becomes diagonal depends on the features of the interaction. Consider this in the same very simple example.

Let the initial state of the system $\mathcal{S}$ be $|\psi\rangle$ from Eq. (1) and the initial state of its environment $\mathcal{E}$ be $|\Phi\rangle$. If the system interacts with the environment during some time, the initial state $|\psi\rangle|\Phi\rangle$ of the total system (composed of $\mathcal{S}$

and $\mathcal{E}$) goes over to the state $U|\psi\rangle|\Phi\rangle$ with some unitary evolution operator U (depending on the interaction). Let the states $|a\rangle$, $|b\rangle$ be conserved by the interaction so that

$$U|a\rangle|\Phi\rangle = |a\rangle|\Phi_a\rangle, \quad U|b\rangle|\Phi\rangle = |b\rangle|\Phi_b\rangle\,. \tag{4}$$

Such an interaction realizes a *measurement* of the system (by its environment) distinguishing between the states $|a\rangle$ or $|b\rangle$. Indeed, observing the state $|\Phi_a\rangle$ or $|\Phi_b\rangle$ of the environment $\mathcal{E}$ provides the information on what of the states $|a\rangle$ or $|b\rangle$ the system $\mathcal{S}$ was in before the interaction (and stays after it). The interaction of this type takes place if the interaction Hamiltonian commutes with the observables $|a\rangle\langle a|$, $|b\rangle\langle b|$.

If the interaction between the subsystems $\mathcal{S}$ and $\mathcal{E}$ satisfies the condition (4), then, owing to the linearity of the operator U, the initial state $|\psi\rangle|\Phi\rangle$ of (1) goes over (after the interaction) to the state

$$|\Psi\rangle = U|\psi\rangle|\Phi\rangle = \alpha|a\rangle|\Phi_a\rangle + \beta|b\rangle|\Phi_b\rangle\,. \tag{5}$$

The state (5) is said to be an *entangled state* of the two subsystems $\mathcal{S}$ and $\mathcal{E}$. This term underlines that the state cannot be presented in the form of the product of state vectors of each of these subsystems. Entanglement of two subsystems leads to decoherence of each of them. This may be shown as follows.

If the state of the compound system $\mathcal{S}+\mathcal{E}$ is presented by the vector $|\Psi\rangle$ (or, what is the same, by the density matrix $|\Psi\rangle\langle\Psi|$), then the state of its subsystem $\mathcal{S}$ is described by the reduced density matrix equal to the trace of $|\Psi\rangle\langle\Psi|$ over the degrees of freedom of the subsystem $\mathcal{E}$:

$$\begin{aligned}\rho &= \mathrm{tr}_\Phi|\Psi\rangle\langle\Psi| \\ &= |\alpha|^2|a\rangle\langle a| + \alpha\beta^*\langle\Phi_b|\Phi_a\rangle|a\rangle\langle b| + \beta\alpha^*\langle\Phi_a|\Phi_b\rangle|b\rangle\langle a| + |\beta|^2|b\rangle\langle b| \\ &= \begin{bmatrix} |\alpha|^2 & \alpha\beta^*\langle\Phi_b|\Phi_a\rangle \\ \beta\alpha^*\langle\Phi_a|\Phi_b\rangle & |\beta|^2 \end{bmatrix}.\end{aligned} \tag{6}$$

Here the states $|\Phi_a\rangle$, $|\Phi_b\rangle$ are taken to be normalized. Besides, the modulus of the scalar product of these states is smaller than unity. Therefore, the off-diagonal matrix elements of the matrix (6) are smaller than those of the matrix (2). This means that a partial decoherence of this system occurred as a result of the interaction with the environment. The complete decoherence occurs if the states $|\Phi_a\rangle$, $|\Phi_b\rangle$ of the environment are orthogonal. This always takes place if the environment is macroscopic and these states are macroscopically distinct. The complete decoherence takes place also in the mesoscopic situation provided these states are orthogonal.

We compared only the initial (before decoherence) and final (after decoherence) states of the system. Typical is the situation when decoherence develops in a very short time.[21] Then there is no reason (at least from practical point of view) to consider time evolution of the system during this short period. If however decoherence is slow, it is important to describe it as a process developing in time. Evidently, this may be described as time dependence of the density matrix of the system. In most cases the process is Markovian, i.e. the state (density matrix) at some moment completely determines the state in all future moments. In this case time dependence of the density matrix may be characterized by a differential equation including the time derivative. This sort of equation was used by M. Marinov[1–3] who added the double-commutator term to the usual von Neumann equation:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}\,[\hat{H},\rho] - \tau\frac{1}{2\hbar^2}\,[\,\hat{H},[\hat{H},\rho]\,] + \mathrm{O}(\tau^2)\,. \tag{7}$$

A more general equation for the density matrix was investigated by G. Lindblad.[7] Such equations are usually called *master equations.*

It is important for our aim that the slow gradual decoherence may be interpreted as a *continuous measurement.* The idea is following. Interaction of a quantum system with its environment changes the state of the environment. As a result, a certain information about the state of the system is contained in the state of the environment. The interaction may therevore be considered as a sort of prolonged (continuous) measurement. We shall see below that this enables one to describe the process in terms of *restricted path integrals.*

The question naturally arises about what are conditions for the decoherence to be slow. We mentioned already that a macroscopic environment leads usually to a very fast decoherence. However decoherence caused by a mesoscopic environment is often slow. This is the case for example in quantum-optical models of measurements.[22] Another example of a slow decoherence may become practically important. It is decoherence in quantum computers. In an ideal case no decoherence at all should occur in a device working as a quantum computer. In practice, decoherence in a quantum computer is inevitable but has to be very slow.

3 Restricted path integrals

Decoherence of a quantum system is caused by an interaction with its environment. Therefore the description of decoherence may be derived from the consideration of a concrete model of the environment and the interaction with it. It is interesting however that such a description may also be obtained without any model, from general principles. This may be achieved with the help of the path-integral formulation of quantum mechanics suggested by R. Feynman[23] and the formalism of *restricted path integrals* developed by the present author[17–19,6] after the idea of Feynman.[24]

In the Feynman's formulation of quantum mechanics the probability amplitude for a system to propagate during a time interval $[t', t'']$ along a path

$$[p, q] = \{\, [p(t), q(t)] \mid t' < t < t'' \,\}$$

in the phase space of this system is equal to

$$U[p, q] = \exp\left[\frac{i}{\hbar}\int_{t'}^{t''} dt\,(p\dot{q} - H(p, q, t))\right]. \tag{8}$$

If, under conditions of the given experiment, there is no way to find out what path is taken by the system, the complete propagator between the points q' and q'' must be calculated as the sum (integral) over all paths $[p, q]$. This leads to the propagator in the form of the *Feynman path integral* in the phase-space representation:

$$U(q'', t'' \,|\, q', t') = \int d\,[p, q]\, \exp\left[\frac{i}{\hbar}\int_{t'}^{t''} dt\,(p\dot{q} - H(p, q, t))\right], \tag{9}$$

where the integral is meant over $[p, q] = [p]\,[q]$ with all paths $[p]$ in the momentum space and those paths $[q]$ in the configuration space which satisfy the boundary conditions $q(t') = q'$, $q(t'') = q''$.

The measure of the path integration (in symbolic form, see details in Refs. 23 and 6) is [a]

$$d[p, q] = \prod_{t=t'}^{t''} \frac{dq(t)\, dp(t)}{2\pi\hbar}\ . \tag{10}$$

The propagator (9) satisfies Schrödinger equation,

$$\left(\frac{d}{dt''} + \frac{i}{\hbar}\hat{H}''\right) U(q'', t'' \,|\, q', t') = 0\,. \tag{11}$$

The two primes of the operator $\hat{H}''$ mean here that this operator acts on the variable q''.

[a] Here a one-dimensional system is considered for simplicity. In case of many degrees of freedom an analogous measure must be taken for each of them.

If a *continuous measurement* is performed during the time interval $[t', t'']$, then the result (readout) α of this measurement contains certain information about what path $[p, q]$ the system propagates along. In this case the path integral has to be restricted in accord with this information. In the simplest case the measurement readout being α means that the path taken by the system lies in a certain subset J_α of paths. Then the integration in the path integral has to be restricted by this subset of paths. In a more general (and more realistic) case the information is expressed by some weight functional $w_\alpha[p, q]$ in the space of paths. Then the path integral has to be taken over all paths but with this functional in the integrand:

$$U_\alpha(q'', t'' \,|\, q', t') = \int d[p, q]\, w_\alpha[p, q] \exp\left[\frac{i}{\hbar}\int_{t'}^{t''} dt\, \left[p\dot{q} - H(p, q, t)\right]\right]. \tag{12}$$

This gives *partial propagators* $U_\alpha(q'', t'' \,|\, q', t')$ in the form of a *restricted path integral*.[18,19,6] The system's evolution is presented then by the set of *partial evolution operators* U_α having the partial propagators as their kernels. Namely, the initial state $|\psi\rangle$ is converted into the state

$$|\psi_\alpha(t'')\rangle = U_\alpha |\psi(t')\rangle \tag{13}$$

in the evolution during the time interval $[t', t'']$ under the condition that the continuous measurement during this interval had been performed and gave the readout α. The same may be expressed in terms of the density matrix:

$$\rho_\alpha(t'') = U_\alpha \rho(t') U_\alpha^\dagger\,. \tag{14}$$

The latter form of the evolution law is applicable also to a mixed initial state.

The state vector (13) and the density matrix (14) are not normalized. Instead, the square norm of the state vector or the trace of the density matrix is the *probability density of the measurement readout* α (assuming the initial state is normalized). Thus,

$$\int_{\mathcal{A}} d\alpha\, \mathrm{tr}\rho_\alpha(t'') \tag{15}$$

is the probability that the measurement readout α belongs to the set $\mathcal{A}$.

The formulas (13) and (14) provide a *selective description* of the evolution which takes into account the readout given by the measurement. If the measurement is performed but its readout is not known, then summation (integration) must be performed over all possible readouts:

$$\rho(t'') = \int d\alpha\, \rho_\alpha(t'') = \int d\alpha\, U_\alpha\, \rho(t')\, U_\alpha^\dagger\,. \tag{16}$$

The measure $d\alpha$ has to be chosen in such a way that the following *generalized unitarity condition* be valid:

$$\int d\alpha\, U_\alpha^\dagger U_\alpha = \mathbf{1}. \tag{17}$$

Then the total density matrix $\rho(t'')$ is normalized provided the initial density matrix $\rho(t')$ is. This guarantees conservation of probability in the evolution under the continuous measurement.

4 Monitoring of an observable

In the preceding section we considered a generic continuous measurement and denoted its readout symbolically by α. Let us consider now a concrete and very important continuous measurement, the *monitoring of an observable* [b] $A = A(p, q, t)$. A readout obtained in the course of the monitoring is presented by the curve $[a] = \{a(t) \,|\, t' < t < t''\}$ so that $a(t)$ is an estimate for the value of A at the time moment t.

A finite precision Δa of the measurement has to be taken into account. This means that the actual value of A at time t may differ from $a(t)$ but not more than by Δa. Therefore the readout denoted as $[a]$ may be in a more adequate manner presented by the *corridor* $J_{[a]}$ of curves $[a']$ close to the curve $[a]$. Namely, $J_{[a]}$ may be the corridor of the width $2\Delta a$ with the curve $[a]$ in its center. Then the partial propagator or partial evolution operator $U_{[a]}$ will be equal to the path integral over the paths $[p, q]$ such that $A\,[p(t), q(t), t\,]$ is a curve lying in the corridor $J_{[a]}$ for each of these paths.

This definition of the partial propagator is palpable but not quite realistic. More realistic would be a *'corridor with fuzzy boundaries'* that may be presented by a weight functional $w_{[a]}[p, q]$. The functional must be close to unity for the curve $A(p(t), q(t), t)$ near $a(t)$, and close to zero for these two curves far from each other. The simplest (and also quite realistic, see Sec. 5) choice of the weight functional is *Gaussian functional*:

$$w_{[a]}[p, q] = \exp\left[-\kappa \int_{t'}^{t''} dt\, \Big(A(p(t), q(t), t) - a(t)\Big)^2\right]. \tag{18}$$

Here κ is a parameter inversely proportional to the 'width' of the Gaussian corridor. Therefore it defines the precision of the measurement. A more

[b] For simplicity we shall consider a single observable, but actually A may be multicomponent with commuting components. The generalization on the monitoring of many non-commuting observables is preliminary discussed in Ref. 6. The definition of path integrals may need further elaboration in this case.

obvious characteristic, the 'width' Δa of the fuzzy corridor may be defined by the formula

$$\kappa = \frac{1}{\Delta a^2 (t'' - t')} \, . \tag{19}$$

It is important that Δa depends on the duration $T = t'' - t'$ of the continuous measurement so that a longer measurement is more precise (for constant κ).

A corridor of paths as well as the corresponding restricted path integral may be called *quantum corridor*. The palpable image of a corridor the path integral is taken over, justifies applying the term 'quantum corridor' to the restricted path integral even in the most general case.

The specific Gaussian form (18) of the weight functional $w_{[a]}$ in case of monitoring enables one to introduce an effective Hamiltonian and the corresponding effective Schrödinger equation for the partial propagator $U_{[a]}$ (and therefore for the 'partial wave function' $|\psi_{[a]}\rangle$). Indeed, usage of Eqs. (18) and (12) gives for a partial propagator $U_{[a]}$ the form of the Feynman path integral

$$U_{[a]}(q'', t'' \,|\, q', t') = \int d\,[p,q] \, \exp\left[\frac{i}{\hbar} \int_{t'}^{t''} dt \left[p\dot{q} - H_{[a]}(p,q,t) \right] \right] , \tag{20}$$

but with the *effective Hamiltonian* $H_{[a]}$ which has to be defined as follows:

$$H_{[a]}(p,q,t) = H(p,q,t) - i\kappa\hbar \left(A(p,q,t) - a(t) \right)^2 . \tag{21}$$

The corresponding quantum operator is

$$\hat{H}_{[a]} = \hat{H} - i\kappa\hbar \left(\hat{A} - a(t) \right)^2 . \tag{22}$$

The partial propagator satisfies therefore the following *effective Schrödinger equation*:

$$\left(\frac{d}{dt''} + \frac{i}{\hbar} \hat{H}''_{[a]} \right) U_{[a]}(q'', t'' \,|\, q', t') = \left(\frac{d}{dt''} + \frac{i}{\hbar} \hat{H}'' + \kappa \left(\hat{A} - a(t) \right)^2 \right) U_{[a]}(q'', t'' \,|\, q', t') = 0 \, . \tag{23}$$

The same equation is satisfied by the wave function evolving according to the partial propagator:

$$\left(\frac{d}{dt} + \frac{i}{\hbar} \hat{H}_{[a]} \right) \psi_{[a]}(q,t) = \left(\frac{d}{dt} + \frac{i}{\hbar} \hat{H} + \kappa \left(\hat{A} - a(t) \right)^2 \right) \psi_{[a]}(q,t) = 0 \, . \tag{24}$$

This equation gives a *selective description* for the continuous monitoring of an observable. Such a description takes into account the measurement readout $[a]$. This enables one to efficiently explore this type of continuous measurements.

Both the probability distribution of the measurement readouts and the evolution of the system given the measurement readout may be found. The procedure is following. One has to fix an initial state $\psi(t')$ and choose various curves $[a]$. For each choice of $[a]$ one has to solve Eq. (24) and find $\psi_{[a]}(t'')$. Then the square norm $\|\psi_{[a]}(t'')\|^2$ is the probability density of the measurement readout $[a]$ for the system initially in the state $\psi(t')$. The time-dependent state $\psi_{[a]}(t)$, $t' < t < t''$, presents the evolution of the system under the condition that the measurement readout is $[a]$. The wave functions $\psi_{[a]}(t)$ may be normalized if necessary.

This gives a selective description of the process of decoherence in the course of monitoring the observable $\hat{A}$. Application of this approach for monitoring various observables in a two-level system may be found in Ref. 6.

Another way to selectively describe decoherence is to make use of *stochastic wave equations*[14–16] (a short account of this approach is given in Ref. 6). This is equivalent[25,26,6] to the effective Schrödinger equation discussed above. The characteristic features of the stochastic equation method are the following: (i) the wave function presenting the system's evolution is normalized; (ii) the stochastic equation is non-linear; (iii) the function presenting influence of the environment has in this approach standard statistics of white noise but no direct physical interpretation. Contrary to this, in our approach the effective Schrödinger equation is linear and the function $a(t)$ (presenting influence of the environment) has the direct interpretation as an *information recorded in the environment about the state of the system.* The latter is important from the conceptual point of view, and we shall discuss this below.

If being not interested in the description being selective, one may go over to the *non-selective description* by integrating over measurement readouts $[a]$. According to the general formula (16), in case of monitoring this gives

$$\rho\,(t'') = \int d[a]\,\rho_{[a]}(t'') = \int d[a]\,U_{[a]}\rho\,(t')\,U^{\dagger}_{[a]}\,. \tag{25}$$

The resulting density matrix $\rho(t)$ may be shown[6] to satisfy the equation

$$\frac{d\rho}{dt} = -\,\frac{i}{\hbar}\,[\hat{H},\rho] - \frac{\kappa}{2}\,[\,\hat{A},[\hat{A},\rho\,]\,]\,, \tag{26}$$

a special case of Lindblad equation.[7] The equation (7) exploited by M. Marinov in his papers[1–3] is evidently a special case of (26) for $\hat{A} = \hat{H}$. More complicated continuous measurements lead in the same way to a more general class of master equations.[6]

In the preceding consideration $a(t)$ was an estimate of the value of the observable A at certain precisely known instant t. Very often this may be accepted as a good approximation. We have seen that this leads to a differential equation (26) for the density matrix. Therefore this approximation is Markovian.

A more precise description of the monitoring may be given in the following way. For $[a]$ being a measurement readout, $a(t)$ is interpreted as an estimate of the value of the observable A averaged over certain time interval containing the moment t. For example it may be the average of A over the time interval of duration τ around moment t. The duration τ is then the *'time resolution' of the measurement.* This type of monitoring may also be formulated and explored in terms of restricted path integrals.[6] This supplies a *non-Markovian approximation* in presentation of monitoring, still in the form of restricted path integrals. The measurement of this type is not local in time and therefore cannot be expressed with the help of a differential equation with time derivatives.

5 Quantum monitoring and quantum Central Limiting Theorem

In the preceding sections a description of a continuous measurement, or continuous decoherence, is derived from the Feynman path-integral formulation of quantum mechanics. Such a model-independent derivation, based only on first principles, has advantages which will be discussed later. However there are people for whom such a derivation seems to be not convincing. This is why a number of concrete models of systems and their environments were explored by usual quantum-mechanical methods[27–29,6] and the results were compared with those obtained in the framework of the restricted-path-integral approach. Coincidence of the results of both approaches was confirmed in each case.

There is one more problem solved in this series of papers. We saw in the preceding consideration that a quantum continuous measurement may be presented by restricted path integrals if a set $\{\alpha\}$ of possible measurement readouts and associated weight functionals $\{w_\alpha\}$ are chosen. In case of observable's monitoring the Gaussian form of weight functionals (18) was suggested. The question naturally arises whether the Gaussian weight functional describes any feasible measurement. Another formulation of the question is whether any environment leads to the decoherence described by the Gaussian weight functional.

The answer to this question is that 'Gaussian decoherence' is not only feasible but also typical. Namely, it arises each time when decoherence is caused by the action of a large number of very weak influences independent of each other. The situation resembles Central Limiting Theorem in probability theory so that a sort of a *quantum analogue of Central Limiting Theorem* seems to be valid. This generalization has not been proven or even precisely formulated, but the consideration of a class of continuous measurements confirmed this suggestion in the following way.

The demonstration of this feature of continuous measurements was given in Refs. 28 and 6. A two-level system was considered which periodically briefly and weakly interacts with a subsidiary device called *'meter'*. Thus, the two-level system undergoes a long series of weak (fuzzy) *observations*. The whole series of observations may be interpreted as a single continuous measurement. Each interaction brings the meter into one of two orthogonal states. The probabilities of them slightly depend on the state of the two-level system. After each interaction the state of the meter is measured and the meter is returned into the initial state to be ready for the next interaction.

Each interaction of the meter with the system gives a vague information about the state of the system and slightly changes the system's state. In the long series of interactions the information about the system as well as the back influence of the interaction onto the system become significant. It is shown that the whole process, including both probability distribution of measurement readouts and the evolution of the system's state, are correctly described by the effective Hamiltonian and the effective Schrödinger equation (24).

The latter shows that the character of continuous measurements or of the precesses of gradual decoherence is universal. A concrete scheme of a continuous measurement (the monitoring of an observable in our case) may be complicated, particularly it may depend on many parameters. However, the final description of the process depends only on the observable A being monitored and the precision κ of the monitoring. It was shown[6] how A and κ are determined by the parameters of the interaction of the two-level system with the meter.

A continuous measurement (realized by a series of weak interactions or in any other way) may be used for obtaining information about quantum processes 'in real time', i.e. without interrupting the processes. Of course, back influence on the process is unavoidable. This can be demonstrated[6] in a simple case where the energy of a two-level system is monitored.

Let a two-level atom undergo the so-called *Rabi oscillation* (i.e. periodically transits from one level to the other and backward) under influence of the resonance laser radiation. If simultaneously the *energy of the atom is monitored*, the picture of Rabi oscillations as developing in time may be derived from the measurement readout. The oscillation is distorted because of the back influence of the measurement. It is interesting how this distortion depends on the strength (resolution) of the monitoring.

If the monitoring is very weak, or very fuzzy (κ is small), the back influence is also very weak so that the Rabi oscillation is not significantly distorted. However the measurement gives in this case a vague information about the state of the atom. In the opposite case when κ is large (strong or highly precise monitoring), the information about the state of the atom is quite definite. However the back influence of the measurement is in this case so strong that the Rabi oscillation is strongly damped. In the limiting case the Rabi oscillation is completely prevented and the system is frozen in the initial state. This phenomenon is known as *quantum Zeno effect* or quantum Zeno paradox.[30,31]

The regimes of monitoring with large and small κ may be called correspondingly *Zeno- and Rabi-regimes*. They are most evidently characterized by the time parameter

$$T_{\rm lr} = \frac{1}{\kappa \Delta E^2} \, , \tag{27}$$

where ΔE is the difference between two energy levels of the atom. The parameter $T_{\rm lr}$ may be called *'level resolution time'*. If monitoring of energy $A = H_0$ (H_0 being a free Hamiltonian of a two-level atom) with the strength κ lasts as long as $T_{\rm lr}$, the system will transit in one of the energy eigenstate (on one of the two energy levels) even if it is in an arbitrary superposition initially. The regime of the measurement depends on the relation between $T_{\rm lr}$ and the period $T_{\rm R}$ of the Rabi oscillation. In case of $T_{\rm lr} \ll T_{\rm R}$ the monitoring is performed in the Zeno regime, while for $T_{\rm lr} \gg T_{\rm R}$ the Rabi regime is realized.

The most interesting is the intermediate regime when $T_{\rm lr}$ is of the order of $T_{\rm R}$. In this case the Rabi oscillation is not completely prevented (although somewhat damped) and the information about the state of the atom (supplied by the monitoring) is more or less definite. This is the only regime in which *transitions between levels may be observed not as instantaneous quantum jumps but as prolonged processes* gradually developing in time. Of course, observing such a process influences it. The result of this back influence is a decrease of the rate.[6]

6 Fundamental character of decoherence

The restricted-path-integral presentation of both continuous measurements and the accompanying process of decoherence quite clearly demonstrates fundamental character of these phenomena. The following features point out to this: (i) theory of continuous measurements may be derived from the first principles; (ii) it reveals the dynamical role of information and (iii) it makes quantum mechanics conceptually close.

(i) The first feature (*derivability from the first principles*) was demonstrated in Sec. 3. The starting point for the theory was quantum mechanics formulated in terms of Feynman path integrals. No additional assumptions or axioms were necessary to construct theory of measurements. The main instrument used for constructing the theory was the ordinary quantum-mechanical rules of dealing with probability amplitudes.

This construction gives in fact a *new type of evolution* described by restricted path integrals instead of unlimited Feynman integrals. From the physical point of view this evolution differs in that it takes into account continuous decoherence. Such an evolution includes *both quantum and classical elements.* Namely, the alternatives α are classical (and this is why they are characterized by probabilities) while each partial propagator or evolution operator U_α presents purely quantum evolution inside the given classical channel α (single paths inside each alternative α may be associated only with probability amplitudes, not probabilities).

Evolution of a quantum system under decoherence (continuous measurement) may be derived from the usual quantum-mechanical consideration of a wider system including not only the system of interest but also its environment (see Sec. 5). However, it is wonderful and exciting that it may also be derived without explicit consideration of the environment, from the first principles of quantum mechanics applied to the system itself. This is an evidence of the fundamental character of the phenomenon.

(ii) One more exciting feature of decoherence is that the *back action of the environment onto a decohering system depends only on the information* about this system recorded in the environment. Different environments interacting with the system in different ways will give the same evolution of this system if the information recorded in them is the same. For example, if the system's coordinates at the successive time moments are recorded in any form in the environment (so that knowledge of the environment's state enables one to calculate these coordinates), then the interaction with the environment may be said to realize the monitoring of this coordinate. The system's evolution is presented then by the path integrals restricted on the corridors in the space of this coordinate (see Sec. 4). The width of the corridors depends on the precision with which the coordinate at each moment may be reconstructed from the state of the environment.

It is this feature that provides a universal character of the evolution of the decohering system (for example Gaussian decoherence in case of monitoring). Of course this is closely related to the previously mentioned derivability of the evolution from the first principles. From practical point of view, this makes it possible to predict the behavior of the decohering system even if details of its interaction with the environment are not known.

(iii) In the long-lasting discussion about conceptual problems of quantum mechanics it was often claimed that quantum mechanics is not conceptually closed. According to this point of view, quantum mechanics describes evolution of a quantum system between any two (instantaneous) measurements but it cannot describe the measurements of the system. What happens with the system in the course of a measurement cannot, according to this opinion, be presented by quantum-mechanical methods and requires a radically different instrument, for example von Neumann's projection postulate. In other words, quantum theory of measurements must be added to quantum mechanics as an independent counterpart of the complete quantum theory.

We saw however that *continuous measurements are naturally described in the framework of quantum mechanics* if the latter is taken in the Feynman path-integral version including theory of probability amplitudes (which was developed in the most complete and consistent form by Feynman). The theory constructed in such a way includes measurements not separately from the quantum evolution (as in the usual theory), but in the non-separable unity with it. Classical elements which are necessary to describe measurements (as has been pointed out already by N. Bohr) need not be added to quantum theory but arise in a natural way inside it (see the above-mentioned remarks about classical character of the alternatives α).

So-called instantaneous measurements (which appear for example in von Neumann's projection postulate) are in fact continuous but very short ones. The concept of a strictly instantaneous measurement may if necessary be derived from theory of continuous measurements in the limit of null duration.[32] Free evolution is of course also a limiting case of the evolution under a continuous measurement. It appears if the strength of the continuous measurement tends to zero. Therefore the situation usually considered in quantum mechanics (periods of free evolution between instantaneous measurements) may be derived from theory of continuous measurements. This completes proof of *closeness of quantum mechanics.*

Thus, quantum theory of continuous measurements is naturally contained in (Feynman form of) quantum mechanics, and this is especially evident in the restricted-path-integral approach to quantum measurements. The price paid for this conceptual advantage is that the *theory deals with open rather than closed systems.* Indeed, besides a Hamiltonian, the system in such a theory is characterized by the set $\{\alpha\}$ of classical alternatives. Each of these alternatives is interpreted as an information about the system recorded in its environment after the interaction with the system is over. Therefore, besides the system itself some environment of this system is assumed to exist and to interact with the system. The system is therefore open. A closed system may be considered as a limiting case when there is only one alternative α with the weight functional w_α identically equal to unity.

In the conventional quantum mechanics one is used to deal with closed systems. Nevertheless theory of open systems is not only more general (as it has been argued above) but also more realistic than theory of closed systems. Indeed, on one hand there is in fact no strictly closed system except total Universe and on the other hand one may consider an arbitrary wide open system. One more advantage of the restricted-path-integral theory of open systems (including decoherence) is that it resolves known quantum-mechanical paradoxes or rather these paradoxes do not arise in such a theory.[33]

7 The role of consciousness in quantum mechanics

As it has been argued above (Sec. 6), the theory of open quantum systems including decoherence and formulated in terms of restricted path integrals is a consistent and logically closed quantum theory. However in some aspect (not at all necessary from the point of view of a physicist) this theory may seem not quite satisfactory. We mean the two rather abstract and in fact metaphysical requirements that are not met by the theory of open systems. This theory is not sufficient 1) if one desires to include the whole Universe as an object of the theory and 2) if one desires to explain how selection of a single measurement readout occurs.

The *whole Universe* may be considered only as a *closed system*, and this is impossible in a theory dealing only with open systems. If the theorist requires that Universe could be considered as one of the objects of his theory, then the theory of decohering open systems is inappropriate. Of course there is no reason in physics that leads to this requirement. All questions which may be correctly formulated in physics, can be answered in the theory of open systems. Therefore a theory of closed systems is not necessary as a physical theory. It may however be considered on philosophical grounds.

The second argument for extending quantum mechanics beyond the theory of decohering open systems is also metaphysical. In the theory of decoherence all possible readouts $\{\alpha\}$ of a measurement performed on an (open) system may be considered, the probability distribution over the set of these alternative readouts calculated and evolution of the system conditioned by any of these readouts found. This is what actually is necessary for a physicist. This is enough to make (probabilistic) predictions for any measurement or to find out the behavior of any (open) system.

What may one need beyond this? For a physicist the answer is: nothing. However from metaphysical (philosophical) point of view it may seem desirable to answer the next question. Even if being able to describe in a probabilistic way all alternative ways of evolution of the open system, one may ask how a concrete single alternative is selected or *what is the reason (mechanism) of selection*.

These metaphysical questions or requirements are not necessary (or not correct) from any practical (physical) point of view. Nevertheless, from the theoretical point of view, it is interesting to consider them, because they lead to a quite novel and exciting line of reasoning. The point is that *both theory of closed systems and theory of selection must include consciousness of an observer* as an object of consideration. This is one of the reasons why unphysical questions of this type have always been discussed and are still under discussion now.[34–37]

The concept of measurement presupposes existence of an observer who reads out the measurement results or can in principle read out them. However, the boundary (so-called *Heisenberg's cut*) between the *measured system* on one hand and the *measuring device plus observer* on the other hand are to some extend arbitrary. A part of the measuring device or even a part of the observer's body may be included in the subsystem called measured system. This shifts the Heisenberg's cut in the direction of the observer.

In the framework of the theory of open decohering systems the measuring device and observer are not explicitly considered (although are assumed to exist). Shifting the Heisenberg's cut to the direction of the observer corresponds in this theory to widening the open system which is under consideration. One may widen this system (measured system) more and more. If however one wish to remain in the framework of the theory of open decohering systems, he must leave something outside the measured system. The part of Universe left outside has to be capable of distinguishing between alternative measurement results. This may be the observer's brain or even some structures in his brain which are responsible for perceiving difference between the alternatives. Important is however that something must be outside the measured system.

On the other hand, if one tries to deal with an actually closed system, one has to include the whole Universe into it. Then the observer together with his brain and everything that may be responsible for perceiving alternatives are parts of the considered system. The theory then has to deal with the observer's consciousness. Any theory of closed systems must include consciousness.

The same may be said about any theory of selection. In the theory of decoherence of open systems the alternatives are considered together with the probability distribution over these alternatives. The mechanism of selection of one of these alternatives is not considered, and the question about this mechanism has to be estimated as incorrect. Physics well does without this mechanism. If however one insists (on some metaphysical grounds) on necessity to explicitly consider the mechanism of selection, then he is obliged to include into consideration everything that may in principle distinguish between the alternatives. Therefore the observer's consciousness (or consciousness' material carrier) which evidently distinguishes between them must be included in the theory with necessity.

Thus, not only a theory of closed systems but also any theory of individual alternatives (theory of selection) must include the observer's consciousness. The most interesting theory of this type is so-called *many-world interpretation of quantum mechanics* put forward by H. Everett.[38–40]

In this theory a closed system (coinciding therefore with the whole Universe) is considered. There is no decoherence in the theory, so that pure state remains pure. If an initial (before the measurement) pure state is a superposition of the states corresponding to the alternative measurement readouts, then the final (after the measurement is over) state is a superposition of macroscopically distinct states (as it is shown in Sec. 2 in the case of a simple model). The specific assumption of Everett (or at least one of its formulations) was that all components of such a superposition are realized but 'in different worlds'. This means that each of these components describes a possible state of Universe including the corresponding state of the observer (observers). All these states are equally real. Any two states from this set describe the two states of Universe corresponding to different measurement results. The observer's state in each of these 'components of the superposition' describes him as perceiving the corresponding measurement readout.

From practical point of view this theory has no advantage. It only provides a different interpretation, but the physical contents of the theory are the same as in the theory of decoherence. However the Everett's theory is interesting because it can shed light on the concept of consciousness.

Note that all alternatives are present in the Everett's theory on equal foot just as in theory of decoherence. However each alternative is now presented by the state of the whole Universe (of the closed system). Besides, the observers's state and specifically the observer's consciousness is a characteristic of the corresponding alternative. In a sense, this is a description of the mechanism of selection.

As the next (also metaphysical) step in the analysis of this theory, one may assume[33] that *the concept of selection is identical to the concept of con-*

sciousness (or, more precisely, the concept of comprehension). To make this identification more clear, one may say that consciousness is ability to see only a single alternative, only a single component in the superposition of macroscopically distinct states. One more formulation: *consciousness is ability to live in a single classical reality* even if many classical realities are included in the superposition. The two very complicated concepts (selection in quantum physics and consciousness in psychology) are then identified and therefore explained in terms of each other: a psychological explanation of the physical phenomenon of selection and a physical explanation of the psychological phenomenon of consciousness.

This may open new prospects both in theory of consciousness and in quantum theory. For example one may put the question whether the observer's consciousness can influence the result of selection.[41,33] It was shown[33] that the positive answer to this question (existence of *'active consciousness'* capable to influence selection) may be considered to be consistent if one is ready to essentially change the methodological principles usually accepted in science. It may be formulated even as *changing the criteria of reality*.

Without going into detail, let us mention that the status of *personal (individual) experience* must become in this case higher than it is usually accepted. An individual experience may in principle be different from what is called 'common experience'. It is only the common experience (but not the individual one) that can be confirmed or refuted by statistics. Usually only the common experience (confirmed by statistics) is accepted as objective reality. In the new methodology (which is necessary if the hypothesis of active consciousness is accepted) the individual experience must also be considered as objective.[33] In fact, this is natural in the framework of the Everett's theory because there are many equally real worlds in this theory (instead of only one real world usually presupposed).

Of course, this change of methodology is radical and hardly acceptable for modern science based upon repeated experiments as the only way to prove something. Therefore, the hypothesis of active consciousness cannot be included into what is now called science. It has to be considered either as a non-scientific concept or as one leading to a qualitatively different (more wide) understanding of what is science. The new science (or the new level of science) may then include some spiritual phenomena which are considered, up to now, as being inconsistent with science. If accepting this line of thinking, one obtains a more wide or more general view of the world than the view given by the modern science. This may give a chance to include in the same framework those spiritual areas which are considered now as being quite distant from each other or even opposite, for example *science and religion*.

8 Conclusion

In the preceding sections a specific view of an important phenomenon of decoherence was surveyed. Let us underline the main points of it.

Decoherence of a quantum system is a process caused by interaction of the system with its environment. In the course of decoherence the state of the system acquires classical features. This manifests itself in the conversion of a pure state into a mixed state in the course of decoherence. This is why decoherence is usually described by a density matrix instead of a wave function (state vector). Typically evolution of the density matrix in the course of decoherence is presented by the differential equation called master equation.

A decohering system must be described by a density matrix (instead of a state vector) if the state of the environment is undetermined. This type of description is called non-selective. If the state of the environment is explicitly taken into account in some way or another, the decohering system may be described by a state vector. Such a description is called selective.

Restricted path integrals enable one to take into account the state of the environment in terms which are characteristic of the decohering system itself. Namely, the influence of the environment onto the decohering system is expressed in this approach in terms of the information about this system which is recorded in the state of the environment. This means that the process of decoherence may be considered as continuous (prolonged in time) measurement of the system.

More concretely, the information about the system recorded in the environment (the measurement readout) may be presented as a quantum corridor, i.e. a set of paths $[p, q]$ in the phase space of the system or, in a more general case, as a functional in the space of paths $[p, q]$. Evolution of the system under the influence of the environment (i.e. under the back influence of the measurement) is presented by the propagator equal to the path integral over the quantum corridor (in general case, the path integral with the corresponding functional in the integrand). The propagator depends on the information recorded in the environment (on the measurement readout). Complete description of the dynamics of the decohering (measured) system is given by a set of these partial propagators corresponding to all possible measurement readouts. In a special case of monitoring an observable, the dynamics of the measured system is described by an effective Schrödinger equation.

The formalism of restricted path integrals or (in a special case) effective Hamiltonians allows one to construct a self-consistent theory of open quantum systems which is fundamental i.e. derivable from the first principles. Quantum mechanics of open systems formulated in this language is logically closed, contains quantum theory of measurements and leads to no paradoxes.

A decohering open system may evolve in various classical channels corresponding to various measurement readouts. Selection of one of these alternative channels is random and may be characterized by the probability distribution. This is why the evolution of an open system is a stochastic process. In the restricted-path-integral approach this process is described by a set of 'underunitary' partial evolution operators instead of a single unitary evolution operator. This approach underlines the dynamical role played by information.

Any theory of closed quantum systems as well as a theory explaining the mechanism of selection should explicitly include consciousness of observers. Such a theory is not necessary for solving those problems which are typical for physics, but it may be motivated by metaphysical arguments. The most interesting attempt to construct a theory of this type is the many-world interpretation of quantum mechanics suggested by Everett. Metaphysical as it is, such a theory nevertheless evokes high interest because its further elaboration may shed light upon the nature of consciousness directly connecting the physical concept of selection and psychological concept of consciousness.

Acknowledgements

The author is obliged to late Yu. A. Golfand for many very interesting discussions on quantum theory of measurements and conceptual problems of quantum mechanics.

References

1. M. S. Marinov, "Description of time evolution of the $K^0 - \bar{K}^0$ system by the relaxation equation for the density matrix," *Pis'ma ZhETF*, **15**, 677 (1972) [*Sov. Phys. JETP Lett.* **15**, 479 (1972)].
2. M. S. Marinov, "Non-Hamiltonian theory of neutral K-mesons," *Yad. Fiz.* **19**, 350 (1974) [*Sov. J. Nucl. Phys.* **19**, 173 (1974)].
3. M. S. Marinov, "A quantum theory with possible leakage of information," *Nucl. Phys.* B **253**, 609–620 (1985).
4. D. Giulini, E. Joos, C. Kiefer, J. Kupsch, I.-O. Stamatescu, and H. D. Zeh, eds. *Decoherence and the Appearance of a Classical World in Quantum Theory* (Springer, 1996).
5. M. B. Mensky, "Decoherence and the theory of continuous quantum measurements," *Phys. Uspekhi*, **41**, 923–940 (1998) [quant-ph/9812017].
6. M. B. Mensky, *Quantum Measurements and Decoherence. Models and Phenomenology* (Kluwer Academic Publishers, Dordrecht, Boston, and London, 2000) [Russian translation: Moscow, Fizmatlit, 2001].

7. G. Lindblad, "On the generators of quantum dynamical semigroups," *Commun. Math. Phys.* **48**, 119–130 (1976).
8. R. B. Griffiths, "Consistent histories and the interpretation of quantum mechanics," *Journal Stat. Phys.* **36**, 219–272 (1984).
9. M. Gell-Mann and J. B. Hartle, "Quantum mechanics in the light of quantum cosmology" in *Complexity, Entropy, and the Physics of Information*, ed. W. H. Zurek, 425–458 (Addison-Wesley, Reading, 1990).
10. R. Omnès, "From Hilbert space to common sense: A synthesis of recent progress in the interpretation of quantum mechanics," *Ann. Phys.* **201**, 354–447 (1990).
11. M. Gell-Mann and J. Hartle, "Classical equations for quantum systems," *Phys. Rev.* D **47**, 3345 (1993).
12. R. Omnès, *The Interpretation of Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1994).
13. H. Carmichael, *An Open Systems Approach to Quantum Optics*, Lectures presented at the Universite Libre de Bruxelles (Springer, 1993)
14. N. Gisin, "Stochastic quantum dynamics and relativity," *Helv. Phys. Acta*, **62**, 363–371 (1989).
15. L. Diosi, "Models for universal reduction of macroscopic quantum fluctuations," *Phys. Rev.* A **40**, 1165–1174 (1989).
16. V. P. Belavkin, "A new wave equation for a continuous nondemolition measurement," *Phys. Lett.* A **140**, 355–358 (1989).
17. M. B. Mensky, "Quantum restrictions for continuous observation of an oscillator," *Phys. Rev.* D **20**, 384–387 (1979).
18. M. B. Mensky, "Quantum restrictions on measurability of parameters of motion for a macroscopic oscillator," *Sov. Phys. JETP*, **50**, 667–674 (1979).
19. M. B. Mensky, *Continuous Quantum Measurements and Path Integrals* (IOP Publishing, Bristol and Philadelphia, 1993).
20. M. B. Mensky, "Continuously measured systems, path integrals and information," *J. Theor. Phys.* **37**, 273–280 (1998).
21. W. H. Zurek, "Decoherence and the transition from quantum to classical," *Physics Today*, **44**, 36–44 (1991).
22. J. M. Raimond, M. Brune, and S. Haroche, "Reversible decoherence of a mesoscopic superposition of field states," *Phys. Rev. Lett.* **79**, 1964 (1997).
23. R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill Book Company, New York, 1965).
24. R. P. Feynman,"Space-time approach to non-relativistic quantum mechanics," *Reviews of Modern Physics*, **20**, 367 (1948).

25. C. Presilla, R. Onofrio, and U. Tambini, "Measurement quantum mechanics and experiments on quantum Zeno effect," *Ann. Phys.* **248**, 95–121 (1996).
26. S. Albeverio, V. N. Kolokol'tsov, and O. G. Smolyanov, "Continuous quantum measurement: Local and global approaches," *Reviews in Math. Phys.* **9**, 907–920 (1997).
27. A. Konetchnyi, M. B. Mensky, and V. Namiot, "The physical model for monitoring position of a quantum particle," *Phys. Lett.* A **177**, 283–289 (1993).
28. J. Audretsch, M. B. Mensky, and V. A. Namiot, "How to visualize a quantum transition of a single atom," *Phys. Lett.* A **237**, 1–9 (1997).
29. M. B. Mensky, "Finite resolution of time in continuous measurements: Phenomenology and the model," *Phys. Lett.* A **231**, 1–8 (1997).
30. B. Misra and E. C. G. Sudarshan, "The Zeno's paradox in quantum theory," *Journ. Math. Phys.* **18**, 756–763 (1977).
31. C. B. Chiu, E. C. G. Sudarshan, and B. Misra, "Time evolution of unstable quantum states and a resolution of Zeno's paradox," *Phys. Rev.* D **16**, 520–529 (1977).
32. T. Calarco, "Impulsive quantum measurements – restricted path-integral vs von-Neumann collapse," *Nuovo Cimento* B **110**, 1451–1461 (1995).
33. M. B. Mensky, "Quantum mechanics: New experiments, new applications and new formulations of old questions," *Phys. Uspekhi*, **43**, 585–600 (2000).
34. E. Squires, *The Mystery of the Quantum World*, Second edition, (IOP Publishing, Bristol and Philadelphia, 1994).
35. R. Penrose, *Shadows of the Mind. A Search for the Missing Science of Consciousness* (Oxford University Press, Oxford, 1994).
36. M. Lockwood, "'Many minds' interpretations of quantum mechanics," *Brit. J. Phil. Sci.* **47**, 159–188 (1996).
37. A. Whitaker, "Many minds and single mind interpretations of quantum theory," in *Decoherence: Theoretical, Experimental, and Conceptual Problems*, eds. Ph. Blanchard, D. Giulini, E. Joos, C. Kiefer, and I.-O. Stamatescu (Springer, Berlin, 2000) pp. 299–307.
38. H. Everett III, "Relative state" formulation of quantum mechanics," *Rev. Mod. Phys.* **29**, 454–462 (1957) [reprinted in Ref. 42].
39. B. S. DeWitt and N. Graham, eds. *The Many-Worlds Interpretation of Quantum Mechanics* (Princeton University Press, Princeton, 1973).

40. H. Everett, "The theory of the universal wave function" in *The Many-Worlds Interpretation of Quantum Mechanics*, eds. B. S. DeWitt and N. Graham (Princeton University Press, Princeton, 1973).
41. E. P. Wigner, "Mind-body question" in *The Scientist Speculates*, ed. L.G. Good (Heinmann, London, 1961) pp. 284-302 [reprinted in Ref. 42].
42. J. A. Wheeler and W. H. Zurek, eds. *Quantum Theory and Measurement* (Princeton University Press, Princeton, 1983).