TOPICS ON THE MAGNETIC MONOPOLES IN QCD

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We overview the issue of the monopole condensation in non-Abelian theories. We emphasize the simplicity of the (lattice) experimental picture and some difficulties of its interpretation in theoretical terms. Some features of the polymer picture of the monopole cluster and of the corresponding field theory are discussed.

Contents

1 Well understood monopoles

There are actually quite a few types of monopoles discussed: say, Dirac monopoles, non-Abelian monopoles, lattice monopoles. Their theoretical status is in fact very different. Let us begin with the cases well understood theoretically (and we confine ourselves to pure gauge theories, without Higgs fields). Emphasizing, of course, the points relevant to the later discussion of the "realistic case" of the non-Abelian monopoles.

1.1 Compact $U(1)$

The show case of the monopole condensation is the compact $U(1)$. As is noted first by Polyakov[1] the Dirac string does not cost any energy because of the compactness of the lattice QED. The action is defined as a sum over the plaquette actions and for the latter, roughly speaking one has,

$$S_{plaq} \;=\; -\frac{1}{2e^2}\mathrm{Re}\left(\mathrm{e}^{i\oint A_\mu dx_\mu} - 1\right)\,, \tag{1}$$

where the contour integral is over the plaquette boundary. For the soft fields (on the lattice spacing scale) the action is proportional to $F^2_{\mu\nu}$ and reduces to the standard one. While for the Dirac string the action vanishes provided that the Dirac quantization condition, $e\,g_m = 2\pi n$ is satisfied.

As for the self energy due to the magnetic field, it is linearly divergent at small distances:

$$M_{mon} \;=\; \frac{1}{8\pi}\int \mathbf{B}^2 d^3r \;\sim\; \frac{1}{8e^2}\frac{1}{a}\,, \tag{2}$$

where a is the lattice spacing, e is the electric charge and the magnetic charge is[a] $g_m = 2\pi/e$. Thus, the monopoles are still infinitely heavy and, at first sight, this precludes any condensation since the probability to find a monopole trajectory of the length L is suppressed as

$$\exp(-S) \;=\; \exp\left(-\frac{c}{e^2}\cdot\frac{L}{a}\right)\,. \tag{3}$$

Note that the constant c depends on the details of the lattice regularization but can be found explicitly for any concrete case.

However there is an exponentially large enhancement factor due to the entropy. Namely, the trajectory of the length L can be realized on a cubic lattice in $N_L = 7^{L/a}$ various ways. Here 7 stands for the number of directions

[a] The notation g is reserved for the non-Abelian coupling, the magnetic coupling is denoted as g_m.

in 4D in which the monopole can go at each step. The eight direction looks exactly backward with respect to the trajectory and if this eight direction is chosen the trajectory would be cancelled against the previous step. The entropy factor,

$$N_L \;=\; \exp\left(\ln 7 \cdot \frac{L}{a}\right)\,, \tag{4}$$

cancels the suppression due to the action (3) if the coupling e^2 satisfies the condition

$$e^2_{crit} \;=\; c/\ln 7 \;\approx\; 1\,, \tag{5}$$

where we quote the numerical value of e^2_{crit} for the Wilson action and cubic lattice. At e^2_{crit} any monopole trajectory length L is allowed and the monopoles condense.

This simple theory works within about one percent as far as the value of e^2_{crit} is concerned.[2] Note that the energy-entropy balance above does not account for interaction with the neighbors.

1.2 Non-Abelian monopoles in the classical limit

Consider now the non-Abelian theory with the Lagrangian

$$L \;=\; -\frac{1}{4g^2}(F^a_{\mu\nu})^2\,,$$

where for simplicity we confine ourselves to the $SU(2)$ case. Also, we will not include quarks.

On the classical level, theory of non-Abelian monopoles turns to be extremely simple as well (for review and further references see, e.g. Refs. 3, 4). First, there are no specific non-Abelian classical solutions and all the monopoles are gauge rotations of the Dirac monopoles with the corresponding magnetic charge. Moreover, presence of the massless particles of spin 1, gluons, affects the monopoles drastically. Indeed, the $U(1)$ charge of the gluons is $e_{gl} = g$. Thus the minimal magnetic charge allowed by the Dirac quantization condition for the gluons is

$$(g_m)_{min} \;=\; \frac{2\pi}{g}\,. \tag{6}$$

The Wilson action however is formulated in terms of the adjoint representation. For the quarks the $U(1)$ charge is $e_q = g/2$ and the minimal magnetic charge is twice as big as (6). Thus, the action (1) is not vanishing but, to the contrary, is equal to its maximal possible value, $S^{string}_{plaq} = 1/2g^2$. In the continuum limit, the string energy per unit length is quadratically divergent in the ultraviolet, $E_{string} \sim (L/a^2)$.

As a result, the monopoles with the charge $Q_m = 1$ are infinitely heavy. Not only because of the energy of the radial field (see Eq. (2)), but even more so because of the energy of the Dirac string attached. Therefore the $Q_m = 1$ monopoles can be introduced only as external probes of the vacuum of the non-Abelian theory (here Q_m is the magnetic charge in units of the minimal magnetic charge (6)) Moreover, there is no space for dynamical quarks since the Dirac string would be visible to the quarks.

All other monopoles with other magnetic charges, $|Q_m| \neq 1$, are unstable: charged gluons fall onto the center.

There is a remarkably (conceptionally) simple way to introduce the external $Q_m = 1$ monopoles on the lattice through the 't Hooft loop.[5] Namely, the monopoles are visualized as end-points of the corresponding Dirac strings which in turn are defined as piercing negative plaquettes. The trick to introduce the negative plaquettes on the lattice is to formally change the sign of the square of the coupling g^2 on a manifold of plaquettes. Then these plaquettes become negative in the limit $|g^2| \to 0$.

Proceeding to more detailed definitions, the 't Hooft loop is formulated[6] in terms of the action

$$S(\beta, -\beta) = \beta \sum_{p \notin M} \mathrm{Tr}\, U_p - \beta \sum_{p \in M} \mathrm{Tr}\, U_p \,, \tag{7}$$

where M is a manifold which is dual to the surface spanned on the monopole world-line j. Introducing the corresponding partition function, $Z(\beta, -\beta)$ and considering a planar rectangular $T \times R$, $T \gg R$ contour j one can define

$$V_{m\bar{m}}(R) \;\equiv\; - \frac{1}{T} \ln \frac{Z(\beta, -\beta)}{Z(\beta, \beta)} \,. \tag{8}$$

By the analogy with expectation value of the Wilson loop the quantity $V_{m\bar{m}}(R)$ is referred to as monopole-antimonopole, or heavy monopole potential.

1.3 Heavy monopole potential at short distances

There is no difficulty to predict the heavy monopole potential at short distances. Indeed, as is mentioned above, all the non-Abelian monopoles classically are in fact the same Abelian monopoles. Therefore, classically the potential is:

$$V_{m\bar{m}}(r) \;=\; - \frac{\pi}{g^2} \frac{1}{r} \,. \tag{9}$$

The classical expression is a reliable zero-order approximation at short distances, $r\Lambda_{QCD} \ll 1$. Moreover, the effect of the quantum correction is also fixed on general grounds: the coupling g^2 is to be replaced by its running counterpart $g^2(r)$. Thus:

$$\lim_{r\to 0} V_{m\bar{m}}(r) \;=\; -\,\frac{\pi}{g^2(r)}\frac{1}{r}\,. \tag{10}$$

The result seems absolutely safe theoretically. Its actual derivation is still full of paradoxes, for a review see Ref. 4.

Equation (10) reveals a double-face nature of the heavy, $g_m = 1$ monopoles. On one hand, they are Abelian objects as is testified to by the overall coefficient in front of $1/r$. On the other hand, the full non-Abelian gluon interactions is responsible for the renormalization.

In other words, Eq. (10) poses a paradox which theory should resolve since it concerns short distances in QCD.[8] Namely, the $1/r$ behavior implies one-gluon exchange at short distances. Indeed, Eq. (10) holds down to academically small distances so that no language of "effective infrared QCD" can apply. On the other hand, one-gluon exchange implies change of the monopole color. Which is not possible since the monopoles are in fact Abelian-like and there are no monopoles belonging to a representation of $SU(2)$.

2 Symmetries in the presence of magnetic charges

2.1 Short distances: $U(1)_{el} \times U(1)_{magn}$, $SU(2)_{color} \times U(1)_{magn}$

The paradox would look absolutely unresolvable if it were not so that there existed experience of dealing with somewhat similar paradoxes in case of QED. Indeed, already in the QED case once we introduce both electric and magnetic charges there are in fact two kinds of charges which cannot annihilate each other. Which means that we do not have simply a $U(1)$ symmetry but a product of two $U(1)^{'}$s:

$$U(1) \;\to\; U(1)_{el} \times U(1)_{magn} \tag{11}$$

instead. Moreover, both types of charges are clearly separated only as far as they are at rest. Once there is relative motion, the magnetic and electric charges interact according to the laws of the classical electrodynamics. In the quantum field theory language, the interaction is due to an exchange of one and the same gauge boson, that is photon.

Generically, a dual gauge boson is understood as a field interacting with the magnetic current j_m:

$$L_{int} \;=\; Q_m B \cdot j_m \quad . \tag{12}$$

One of our basic points is that the field B should be treated as fundamental in gluodynamics since the external monopoles introduced via the 't Hooft loop are point like in the continuum limit, see above. A well known example of a theoretical framework to introduce a dual gauge boson is the Zwanziger Lagrangian describing the $U(1)_e \times U(1)_{magn}$ electrodynamics.[7] Formally one introduces two vector fields, the "standard" photon A_μ and the dual photon B_μ, so that $L_{int} = Q_e A \cdot j_e + Q_m B \cdot j_m$. However, the number of the degrees of freedom is not changed since there is a constraint that the field strength tensor constructed on the potential A coincides with the dual field strength tensor constructed on the potential B_μ. More precisely:

$$m_\mu F_{\mu\nu}(B) \;=\; m_\mu {}^* F_{\mu\nu}(A) \tag{13}$$

where m_μ is an arbitrary space-like vector. The choice of the vector m_μ is a kind of new gauge freedom. Physically, m_μ is the vector directed along the Dirac strings. Explicitly:

$$\begin{aligned} L_{Zw}(A,B) \;=\; & \frac{1}{2}\,(m\cdot[\partial\wedge A])^2 + \frac{1}{2}\,(m\cdot[\partial\wedge B])^2 \\ & + \frac{i}{2}\,(m\cdot[\partial\wedge A])(m\cdot {}^*[\partial\wedge B]) \\ & - \frac{i}{2}\,(m\cdot[\partial\wedge B])(m\cdot {}^*[\partial\wedge A]) + i\, j_e\cdot A + i\, j_m \cdot B\,, \end{aligned} \tag{14}$$

where j_e, j_m are electric and magnetic currents, respectively, m_μ is a constant vector, $m^2 = 1$ and

$$[A\wedge B]_{\mu\nu} = A_\mu B_\nu - A_\nu B_\mu\,, \qquad (m\cdot[A\wedge B])_\mu = m_\nu [A\wedge B]_{\mu\nu}\,,$$

$${}^*[A\wedge B]_{\mu\nu} \;=\; \frac{1}{2}\,\varepsilon_{\mu\nu\lambda\rho}\,[A\wedge B]_{\lambda\rho}.$$

Similarly, upon introduction of the (heavy) monopoles the symmetry of the gluodynamics becomes:[8]

$$SU(2) \;\to\; SU(2)_{color} \times U(1)_{magn} \tag{15}$$

where the corresponding groups are used for classification of the charges, non-Abelian and magnetic, respectively. The number of gauge fields (gluons) is of course not increased. The way out of this apparent paradox is a generalization of the constraint (13):

$$m_\mu F_{\mu\nu}(B) \;=\; m_\mu {}^*(n^a F^a_{\mu\nu}(A)) \tag{16}$$

where $F^a_{\mu\nu}$ is the non-Abelian field strength tensor and n^a is an arbitrary vector in the color space. Again, the choice of n^a is a matter of gauge fixing and the physical results do not depend on this choice. The vector n^a fixes the direction in the color space of the magnetic fields transported along the Dirac strings attached to the monopoles.

One can derive a Zwanziger-type Lagrangian which ensures the validity of the constraint (16).[8] This formulation allows then to make predictions for the heavy monopole potential (10). In particular, at short distances the potential can be derived now from the first principles. However, there is a specific problem that standard Feynman rules do not respect the Dirac veto for virtual particles and they interact with the string. Thus, special rules of UV regularization are to be introduce to remove these effects.[8]

To summarize, introduction of the monopoles extends the classification group to $SU(2)_{color} \times U(1)_{magn}$. The choice of a particular gauge boson which is shared between the groups is a matter of gauge fixing which in a way violates the $SU(2)$.

2.2 Spontaneous breaking of the magnetic $U(1)$

At larger distances, there are further points related to the symmetry of problem which should be discussed. Let us begin with the fundamental (point-like) monopoles introduced via the 't Hooft loop. The monopoles are in fact Z_2 monopoles, so that two monopoles of "the same charge" can annihilate each other. Classically, the Z_2 monopoles interact still as $U(1)$ monopoles.[8] If one starts from short distances then the first sign of the Z_2 nature of the monopoles is a possible linear correction to the monopole potential due to a quantum transition between the states with magnetic charge $Q_M = 0$ (the ground state) and $Q_M = 2$ (an excited state):[11]

$$\delta V_{\bar{m}m} \sim M_{0,2} \frac{1}{E_0 - E_2} M_{2,0} \sim g^2 r \,. \tag{17}$$

Unfortunately, little can be said about the corresponding matrix elements $M_{0,2}$. At larger distances, the differences between Z_2 and $U(1)$ monopoles is even more open question. We will ignore this problem and stick to the $U(1)$ description of the monopoles.

At large distances it is also natural to expect that the monopoles condense and the magnetic $U(1)$ is spontaneously broken. Note that no breaking of the (global) $SU(2)$ is introduced at this point as far as the group is indeed $SU(2)_{color} \times U(1)_{magn}$. Moreover since the $U(1)$ boson is actually one of the same $SU(2)$ gluons which interact with quarks, acquiring the mass for the $U(1)$

boson implies confinement of the quarks. In the sense, that forming a tube-like structure for the quarks field is the only way to satisfy the Gauss law.

On the theoretical side, the symmetries of the Lagrangian are becoming difficult to implement in practical calculations once the spontaneous breaking of the magnetic $U(1)$ is introduced. The problems can be illustrated on the example of the Zwanziger Lagrangian (13) for ordinary QED. Indeed, let us assume that a charged scalar field acquires a nonvanishing vacuum expectation value or, even simpler, a mass term $\frac{m_V^2}{2}A_\mu^2$ is added to the Lagrangian. Then a straightforward diagonalization of the bilinear terms in the Lagrangian results in the following propagators of A- and B-fields (see, e.g., Ref. 12):

$$\langle B_\mu B_\nu \rangle(k) = \frac{1}{k^2+m_V^2}\left(\delta_{\mu\nu} + \frac{m_V^2}{(km)^2}(\delta_{\mu\nu} - m_\mu m_\nu) + \cdots\right), \tag{18}$$

$$\langle A_\mu A_\nu \rangle(k) = \frac{1}{k^2+m_V^2}(\delta_{\mu\nu} + \cdots) \tag{19}$$

where the dots stand for terms proportional to k_μ and which can eventually be omitted because of the current conservation. If we evaluate the interaction energy of a monopole pair due to the (massive) photon exchange then the m_μ dependence does not drop off. Also, the double pole in (km) causes infrared problems.

These inconsistencies are not a consequences of a trivial mistake but deeply rooted in the formalism. The point is that the independence on the Dirac strings assumes that the Dirac veto is observed. However, if the charges are condensed, then the strings are "everywhere" and this results in the inconsistencies of the propagator (18). The difficulty was resolved only recently[13] and only in case of static sources (or $k_0 = 0$).

3 Monopoles, as they are seen

3.1 Monopole condensation in non-Abelian case: expectations

If we try to adjust the lessons from the compact $U(1)$ to the non-Abelian case we run into painful questions. The only good news, to begin with, is that all the $U(1)$ subgroups of $SU(2)$ are indeed compact, this is no problem.

Let us therefore try to work out a simple dynamical picture for monopole condensation. Dynamics of any subgroup of the $SU(2)$ is governed by the same running coupling $g^2(r)$. Then – we would conclude – starting from small size lattices we would not see monopoles because $g^2(a)$ falls below e^2_{crit}. However, we could hope that going to a coarser lattice a la Wilson we could come to the point where $g^2(a^2) \approx e^2_{crit}$. Then one could hope to apply the entropy-energy balance which works so well in case of the compact $U(1)$.

The first shock comes if we try to estimate, what the "small a" means numerically, in the present context. Now, the $U(1)$ critical coupling is well known, $e^2_{crit} \sim 1$. In the QCD case we can rewrite the condition (5) as a condition on the critical scale. In the realistic case we have at the LEP energies $E^2 \sim (100 \text{ GeV})^2$, $\alpha \approx 0.1$, $g \approx 1$. Then

$$M_{crit} \;\sim\; 100 \text{ GeV} \tag{20}$$

and, remarkably enough, we are getting rather weak interactions scale than $\sim \Lambda_{QCD}$. On the other hand the lattice data give another estimation of M_{crit}. It is well known that in $SU(3)$ gluodynamics $\beta = 6$ corresponds to the lattice spacing $a \approx 0.1$ fm and the scale is:

$$M_{crit} \;\sim\; 2 \text{ GeV}\,. \tag{21}$$

In other words, according to the first estimate, for any presently available lattice we are deep in the "strong-coupling" region despite of the asymptotic freedom. Then, naively, we could expect that approximating the total energy by self-energy does not work, the monopole population is very dense so that the interaction energy is comparable to the self energy. Moreover, the scale 100 GeV looks rather as a headache than an enlightenment since there is no independent evidence for relevance of such a mass scale to QCD.

This formidable perspective is balanced only by the doubts that the monopoles are at all relevant. The point is that intrinsically the monopoles are a $U(1)$ object and there is no unique way to choose the $U(1)$ subgroup from the $SU(2)$.

Mostly, monopoles are defined in the non-Abelian case as pure topological objects,[10] with no direct relation to the full non-Abelian action. Moreover, it appears obvious that, no matter which $U(1)$ we choose, the minimal action for such a configuration should collapse to zero. At least, on the classical level so.

There is no general proof of this conjecture but as a support for this let us quote the solution for an open Dirac string which costs no non-Abelian action at all.[8] One can show[8] such a configuration is generated from the vacuum by the following gauge rotation matrix:

$$\Omega \;=\; \begin{pmatrix} \mathrm{e}^{i\varphi}\sqrt{A_D} & \sqrt{1-A_D} \\ -\sqrt{1-A_D} & e^{-i\varphi}\sqrt{A_D} \end{pmatrix}, \tag{22}$$

where φ is the angle of rotation around the axis connecting the monopoles and A_D is the $U(1)$ potential representing pure Abelian monopole – antimonopole pair:

$$A_\mu dx_\mu \;=\; \frac{1}{2}\left(\frac{z_+}{r_+}-\frac{z_-}{r_-}\right)d\varphi \;\equiv\; A_D(z,\rho)d\varphi\,, \tag{23}$$

where $z_\pm \;=\; z \pm R/2$, $\rho^2 \;=\; x^2+y^2$, $r^2_\pm \;=\; z^2_\pm+\rho^2$. Note that the action associated with the Dirac string is considered in this case zero, in accordance with the lattice version of the theory (for details see Ref. 8).

To summarize, we are failing to work out a simple dynamical picture mainly because we cannot specify reasonable monopole-like configurations. Our problem is that, first, the action is too low to match ln 7. To fight this, we would rather argue that there is no such object as point-like monopole at all so that the entropy is counted also wrong. But then nothing is left of the idea of the monopole condensation in the non-Abelian case.

3.2 Monopole dominance

On the background of the theoretical turmoil, the data on the monopoles indicate a very simple and solid picture. We will constrain ourselves to the monopoles in the so called Maximal Abelian gauge and the related projection (MAP). We just mention some facts, a review and further references can be found, e.g., in Ref. 14.

Since the monopoles of the non-Abelian theory are expected to actually be $U(1)$ objects one first uses the gauge freedom to bring the non-Abelian fields as close to the Abelian ones as possible. The gauge is defined by maximization of a functional which in the continuum limit corresponds to $R(\hat{A})$ where

$$R(\hat{A}) \;=\; -\int d^4x\left[(A^1_\mu)^2+(A^2_\mu)^2\right] \tag{24}$$

where $1,2$ are color indices.

As the next step, one projects the non-Abelian fields generated on the lattice into their Abelian part, essentially, by putting $A^{1,2}\equiv 0$. In this Abelian projection one defines the monopole currents k_μ for each field configuration.

The relation of the monopoles to the confinement is revealed through evaluation of the Wilson loop for the quarks in the fundamental representation. Namely it turns out, first, that the string tension in the Abelian projection is close to the string tension in the original $SU(2)$ theory:[9]

$$\sigma_{U(1)} \;\approx\; \sigma_{SU(2)}\,. \tag{25}$$

Moreover, one can define also the string tension which arises due to the monopoles alone. To this end, one calculates the field created by a monopole current:

$$A_\mu^{mon}(x) = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}\sum_y \Delta^{-1}(x-y)\,\partial_\nu m_{\alpha\beta}[y;k]\,, \tag{26}$$

where Δ^{-1} is the inverse Laplacian, and sums up (numerically) over the Dirac surface, $m[k]$, spanned on the monopole currents k. The resulting string tension is again close to that of the un-projected theory:

$$\sigma_{mon} \;\approx\; \sigma_{SU(2)}\,. \tag{27}$$

It might worth mentioning that these basic features remain also true upon inclusion of the dynamical fermions in $SU(3)$ case (full lattice QCD).[15]

3.3 Invariant properties of the monopoles.

Despite of the apparent gauge-dependence of the monopoles introduced within the MAP, they encode gauge-invariant information. In particular,we would mention two points: scaling of the monopole density and full non-Abelian action associated with the monopoles.

According to the measurements (see Ref. 16 and references therein) the monopole density ρ_{mon} in three-dimensional volume (that is, at any given time) is expressible in the physical units. In other words, the density scales according to the renormgroup as a quantity of dimension 3. Numerically:

$$\rho_{mon} \;=\; 0.65(2)\;(\sigma_{SU(2)})^{3/2}\,. \tag{28}$$

One important remark is in order here. While discussing the monopole density one should distinguish between what is sometimes called ultraviolet (UV) and infrared (IR) clusters.[17] The infrared, or percolating cluster fills in the whole lattice while the UV clusters are short. There is a spectrum of the UV clusters, as a function of their length, while the percolating cluster is in a single copy. The statement on the scaling (28) refer only to the IR cluster. The UV clusters are seemingly a lattice artifact.

Also, upon identification of the monopoles in the Abelian projection, one can measure the non-Abelian action associated with these monopoles. For practical reasons, the measurements refer to the plaquettes closest to the center of the cube containing the monopole. Since the self energy is UV divergent, it might be a reasonable approximation. The importance of such measurements is that we expect that it is the non-Abelian action which enters the energy-entropy balance for the monopoles.

The results of one of the latest measurements of this type are reproduced in Fig. 1 (see Ref. 18). What is plotted here is the average excess of the action on the plaquettes closest to the monopole (monopoles are positioned at the centers of the cubes). The action is the lattice units. In other words, the straightforward continuum would imply divergent mass of the monopole of order $1/a$ if the action of order unit.

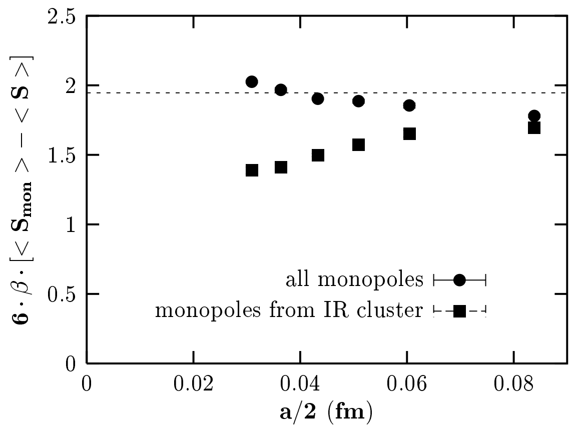

Figure 1: The dependence of the average excess of the action on the plaquettes closest to the monopole, as a function of β.

The conclusions which can be drawn[18] from the measurements are apparently as follows:

i) The IR and UV monopoles are distinguishable through their non-Abelian actions. For the UV monopoles the action is larger, in accordance with the fact that they do not percolate (condense).

ii) The excess of the action for the monopoles falls close to ln 7,

$$S_{mon} \;\sim\; \ln 7 \cdot \frac{L}{a}\,, \tag{29}$$

quite dramatically confirming in this way the reality of the (Abelian-like) energy-entropy balance. Actually evidence to this effect, less direct, was obtained already much earlier, see.[19]

iii) We are talking actually about small distances, by all the standards of QCD. Thus, the action for the IR and UV monopoles are definitely different only at the monopole sizes (radii) about

$$R_{mon} \;\approx\; 0.06 \text{ fm}\,, \tag{30}$$

And even at smallest available distances the monopoles are still very “hot”. The corresponding monopole mass (see Fig. 1) is, roughly, proportional to

$1/a \sim 2$ GeV. We cannot exclude the case $M_{mon} \sim 1/a$ when we approach the continuum limit. The inside region of the monopoles where the asymptotic freedom sets in has not been reached yet even at $a = 0.04 fm$. Which indicates that the surprising estimate (20) might indeed be relevant.

4 Interpretation and phenomenology

4.1 Monopole cluster as a polymer

After learning that the reality of the monopole condensation looks much simpler than our confused expectations we can try to adjust our views. Let us therefore try to summarize, what we have actually learned.

First, our fears that the monopole action would fall too much below the entropy-related ln7 are not realized. To the contrary, at least the first look at Fig. 1 rather suggests that the action is too high. Moreover it has been known from analysis of the monopoles in the Abelian projection that the entropy-energy balance is similar in the non-Abelian and Abelian cases.[19] Which, reinforced now with the data in Fig. 1, means that in the zero approximation $S_{mon}^{phen} \approx \ln 7 \cdot \frac{L}{a}$. Which means, in turn, that the $SU(2)$ monopole action is higher than its Abelian counterpart. Indeed, phenomenological fits suggest:[19]

$$M_{mon} \approx M_{mon}^{Coul} + \text{const}\,, \tag{31}$$

where by M_{mon} we understand the action associated with the monopole trajectory of length L divided by a. Note also that the Coulombic part of the mass, M_{mon}^{Coul} is of order $1/a$.

As it follows from the discussion in section 1, the mass (31) is not so much meaningful and one is to consider the difference between the mass and $\ln 7/a$. The existing data are not precise enough to provide this difference directly. However, it seems natural to speculate that

$$-\mu = \frac{\ln 7}{a} - M_{mon} \sim \Lambda_{QCD}\,. \tag{32}$$

It is a pure assumption but otherwise it would be difficult to understand the scaling of the monopole density. Indeed, if we view the monopole trajectory as a polymer, then there are essentially two entries to describe monopoles, that is the chemical potential and the interaction which is presumably Coulomb-like:

$$S = L\mu + g_m^2 \sum_{a,b}{}' \frac{a^2}{(r_a - r_b)^2}\,, \tag{33}$$

where the primed sum, $\Sigma^{'}_{a,b}$, does not include the self-energy. Thus, if $1/a$ survives in μ it would be very difficult to understand the scaling of the ρ_{mon}.

The central point about the polymer action (33) is that it is written in $SU(2)$ invariant terms alone and, if we can indeed go ahead with such a formulation, we are getting closer to the ideal of describing the magnetic monopoles within $SU(2)_{color} \times U(1)_{magn}$ (see Section 2.2).

4.2 Monopole cluster in the field-theoretical language

The appearance of large masses in the polymer language, $M_{crit} \sim 1/a$ might look quite scary. Moreover, there is no indication to the relevance of such mass to other observables in QCD. Thus, let us address this issue in more detail. To this end, it is desirable to develop a field theoretic language within which the masses can be interpreted more directly. As is well known, the condensation of the monopoles can be described in terms of a (dual) Abelian Higgs model (for review and references see Ref. 14). However, usually this model is formulated within an Abelian-projected action. Now, we would like to continue with the $SU(2)$ invariant description (33).

The transition from the polymer to the field theoretical language is common in the statistical physics (see, e.g., Ref. 20). The first applications to the monopole physics are due to the authors in Ref. 21. Here we, again, emphasize only a few points.

The monopole trajectory represented as a random walk and the corresponding partition function is:

$$Z = \int \mathrm{d}^4 x \sum_{N=1}^{\infty} \frac{1}{N} e^{-\mu N} Z_N(x,x)\,, \tag{34}$$

where μ is the chemical potential and $Z_N(x_0,x_f)$ is the partition function of a polymer broken into N segments:

$$Z_N(x_0,x_f) = \Big[\prod_{i=1}^{N-1} \int \mathrm{d}^4 x_i\Big] \prod_{i=1}^{N} \left[\frac{\delta(|x_i - x_{i-1}| - a)}{2\pi^2 a^3}\right] \exp\Big\{-\sum_{i=1}^{N} gV(x_i)\Big\}\,. \tag{35}$$

This partition function represent a summation over all atoms of the polymer weighted by the Boltzmann factors. The δ–functions in (35) ensure that each bond in the polymer has length a. The starting point of the polymer (35)is x_0 and the ending point is $x_f \equiv x_N$.

In the limit $a \to 0$ the partition function (35) can be treated analogously to a Feynman integral. The crucial step is the a coarse–graining: the N–sized polymer is divided into m units by n atoms ($N = mn$), and the limit is considered when both m and n are large while a and $\sqrt{n}a$ are small. We get,

$$\prod_{i=\nu n}^{(\nu+1)n-1} \frac{1}{2\pi^2 a^3}\delta(|x_i - x_{i+1}| - a) \to \left(\frac{2}{\pi n a^2}\right)^2 \exp\left\{-\frac{2}{n\,a^2}(x_{(\nu+1)n} - x_{\nu n})^2\right\}, \tag{36}$$

where the index i, $i = \nu n \cdots (\nu+1)n - 1$, labels the atoms in ν^{th} unit. The polymer partition function becomes:[21]

$$\begin{aligned} Z_N(x_0, x_f) \quad &= \quad \text{const} \cdot \left[\prod_{\nu=1}^{m-1} \mathrm{d}^4 x\right] \left[\left(\frac{2}{\pi n a^2}\right)^{2m} \exp\left\{\sum_{\nu=1}^{m} \frac{(x_\nu - x_{\nu-1})^2}{n a^2}\right\}\right] \\ &\cdot \exp\left\{-\sum_{\nu=1}^{m} n(\mu + V(x_\nu))\right\}. \end{aligned} \tag{37}$$

The x_i's have been re-labeled so that x_ν is the average value of x in at the coarser cell. Using the variables:

$$s = \frac{1}{8}\,n a^2 \nu\,, \quad \tau = \frac{1}{8} a^2\, N\,, \quad m_0^2 = \frac{8\mu}{a^2}\,, \tag{38}$$

one can rewrite the partition function (34) as

$$Z = \text{const} \cdot \int_0^\infty \frac{\mathrm{d}\tau}{\tau} \int_{x(0)=x(\tau)=x} Dx \, \exp\left\{-\int_0^\tau \left[\frac{1}{4}\dot{x}_\mu^2(s) + m_0^2 + g_0 V(x(s))\right] \mathrm{d}s\right\}. \tag{39}$$

The next step is to rewrite the integral over trajectories $x(\tau)$ as the standard path integral representation for a free scalar field. For us it is important only that the m_0^2 term in the Eq. (39) is becoming the standard mass term in the field theoretical language:

$$\mathcal{Z} = \sum_{M=0}^{\infty} \frac{Z^M}{M!} = \text{const} \cdot \int D\phi \, \exp\left\{-\int \mathrm{d}^4 x \left[(\partial_\mu \phi)^2 + m_0^2\,\phi^2 + g_0 V(x)\phi^2\right]\right\}. \tag{40}$$

The whole machinery can be easily generalized to the case of charged particles (monopoles) with Coulomb-like interactions. Below we assume that we are in the Higgs phase, or that the chemical potential is negative, $\mu < 0$, and the vacuum is stabilized through a repulsive $\lambda\phi^4$ interaction.[21]

Here, it is the relation between ρ_{mon} and $\langle\phi^2\rangle$ which is most important for us. To derive the relation differentiate first the partition function in the polymer representation:

$$\langle L \rangle = \frac{\partial}{\partial \mu} \ln \mathcal{Z}\,. \tag{41}$$

Since the density ρ_{mon} scales:

$$\langle L \rangle = \rho_{mon} \cdot V_4 \,, \tag{42}$$

where V_4 is the 4-volume occupied by the lattice. On the other hand, differentiating the same partition function but in the field theoretical representation (39) with respect to m_0^2 we get the vacuum condensate:

$$\langle \phi^2 \rangle = \frac{\partial}{\partial m_0^2} \ln \mathcal{Z} \,. \tag{43}$$

It is worth emphasizing that in the both cases (41) and (43) we keep only the contribution of the IR monopole cluster corresponding to the condensing Higgs field in the field-theoretic language.

Finally, since the parameters μ and m_0^2 are directly related, see Eq. (38), we get:

$$\langle \phi^2 \rangle \;=\; \frac{1}{8}\, \rho_{mon} \cdot a \,, \tag{44}$$

which is one of our main results [b]. Note that, up to an overall numerical factor, Eq. (44) is quite obvious on the dimensional grounds.

4.3 Naive limit $a \to 0$

Thus, let us assume that the scaling of the monopole density ρ_{mon} continues to be true for smaller lattice spacings as well, at least until we reach the mass scale sensitive to the non-local structure of the monopoles, see discussion above. Then we have the following simple picture:

$$\lim_{a\to 0} m_0^2 \sim \frac{\mu}{a} \to \infty \,, \quad \lim_{a\to 0} \langle \phi^2 \rangle \sim \rho_{mon}\, a \;\to\; 0 \,, \quad \lim_{a\to 0} m_V^2 \sim g^{-2} \rho_{mon}\, a \to 0 \,. \tag{45}$$

It is worth emphasizing that the masses we are discussing here are gauge invariant since we started from the non-Abelian action per unit length. And we see that existence of the huge mass scale (20) might in fact be in no contradiction with the asymptotic freedom. Indeed, only the chemical potential has physical meaning and the scaling of the ρ_{mag} indicates that it is of order Λ_{QCD}. Moreover, the effect of the condensate on the gluon mass goes away as a power of a.

[b] Let us remind the reader that by ρ_{mon} and $\langle \phi^2 \rangle$ we understand in fact the contributions of the IR cluster to these quantities. The v.e.v. $\langle \phi^2 \rangle$ contains the quadratic divergent piece, a^{-2}, due to a perturbative contribution. On the other hand the contribution to the total monopole density from the UV–clusters which should diverge as a^{-3} according to dimensionality arguments. These divergences match each other in Eq. (44).

It is worth emphasizing that Eq. (45) implies that

$$\lim_{a\to 0} m_0^2 \cdot \langle \phi^2 \rangle \;\sim\; \text{const}\,. \tag{46}$$

In other words, the potential energy behaves smoothly as $a \to 0$. And this is, in fact, the most adequate formulation of the emerging picture. It was possible to find the a-dependence for m_0^2 and $\langle \phi^2 \rangle$ separately only because of normalizing the kinetic energy to unit, as usual.

Note that the scaling laws (45) are still consistent with ρ_{mon} = const. Moreover, this seems to be sufficient to ensure the monopole dominance and

$$\lim_{a\to 0} \sigma_{mon} \;\sim\; \text{const}\,, \tag{47}$$

where the monopole string tension is calculated with the use of Eq. (26). Which means in turn that the parameters used to describe the structure of the string within the Abelian projection can be stable in the limit $a \to 0$. Moreover, say,

$$\lim_{a\to 0} (m_V^2)_{Ab.proj.} \;\sim\; \text{const}\,, \tag{48}$$

is in no direct contradiction with (45) since the masses determined in terms of the Abelian-projected action are not directly related to the masses (45) determined in terms of the non-Abelian action.

Thus, the picture which emerges if we start with assumption (32) has some attractive features. However the connection of the string tension with the classical solutions is lost. Moreover, the data in Fig. 1 suggest that the assumption (32) is not valid at a larger than, say, $a_{crit} \sim 0.15$ fm. One might hope for a matching at these distances of the polymer picture with a negative chemical potential μ and of a more phenomenological approach of Refs. 19.

Although little can be said at this time about the $\lambda\phi^4$–type theory corresponding to the monopole "polymer", the whole issue seems to be very interesting. Indeed, it is well known that it is very difficulty, if not impossible, to construct a non-trivial $\lambda\phi^4$ theory and the polymer regularization is commonly discussed in this connection.[27] The scaling law (28) implies that the QCD, when projected onto the properties of the monopole polymer, corresponds to a non-trivial $\lambda\phi^4$–type theory.

4.4 Phenomenology

It is quite clear that the theory of the monopole condensation is far from being complete and numerical predictions are difficult to make. Indeed, we discussed mostly symmetry properties. Nevertheless, let us try to formulate some consequences grouping them into three categories:

(i) We speculated that the polymer formulation (32) encodes the gauge invariant information on the monopoles in the most direct way. Therefore, studies of the monopole trajectories (IR cluster) might be most important. One expects:

a) The monopole trajectories are random walk for any a in the sense that there is no correlation between the vectors tangent to the monopole trajectory;

b) monopole density scales, $\rho_{mon} = \text{const}$ and is independent of ϵ at least as far as the monopole action exceeds the average in the lattice units (and not in Λ^4_{QCD});

c) as is known (see, e.g., Ref. 23) the monopole trajectories intersect. It is natural to speculate that the distance between the self-intersections also scales, reflecting the scaling of the potential energy;

d) the intersections correspond in the field theoretical language to the $\lambda\phi^4$ interaction:

$$V(\phi) \;=\; -\, m_0^2\phi^2 + \lambda\,\phi^4 \tag{49}$$

As we argued, one expects that the potential energy is a-independent. This would imply that the effective scalar mass defined in terms of the second derivative of the potential at the minimum is also a-independent. Which could be checked through measurements.

(ii) We argued that the actual symmetry, in the presence of the magnetic charges, is $SU(2)_{color} \times U(1)_{magn}$. However, direct use of the Zwanziger-type Lagrangian (see Eq. (16)) is not possible already in the $U(1)$ case, see subsection 2.2. Thus, we can rely on the symmetry considerations alone. Then:

a) effective Higgs-like theories introduce dimension d=2 condensate. The resulting leading "effective" correction to the original Lagrangian which is consistent with the symmetry, is the gluon mass. Moreover, generation of the effective mass is a prerequisite of the confinement. There exist independent evidence supporting introduction of the corresponding power correction.[24]

b) Explanation of the Casimir scaling is a challenge to any phenomenological model of the confinement.[25] Within our approach, we cannot directly evaluate the string tension. Indeed, even for the fundamental monopoles at short distances one should account for the full non-Abelian interaction to account for the running of the coupling. Generally speaking, the non-Abelian nature should be even more manifest in large-distance interaction. However, there is a phenomenological observation that the string tension for quarks in the fundamental representation one can use pure Abelian interaction (26). It is quite natural to assume that the same will be true for quarks with the highest value of T_3.[26] Because of the intrinsic $SU(2)$ invariance it should be then true for the invariant string tension $\sigma_{SU(2)}$. Phenomenologically one can explain existing data on the Casimir scaling along these lines.[26]

(iii) The picture of the monopoles dominated vacuum suggests the so called two-step QCD (see the last paper[24] and references therein). Namely, even for non-perturbative fluctuations we may have rather hard fields which, however, are not related directly to the confinement and/or resonance properties. In particular, the condensate $\langle G^2 \rangle_{non\text{-}pert}$ receives the following contribution from the monopoles:

$$\langle G^2 \rangle_{mon} \sim \frac{\rho_{mon}}{a} \tag{50}$$

where a is the lattice spacing and one can use the estimate (50) as long as the inside region of the monopole is not reached.

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