Your Name: ______________________________

MATH 4242: APPLIED LINEAR ALGEBRA
SAMPLE MIDTERM TEST I

You may not use notes, books, etc. Only the exam paper, a pencil or pen may be kept on your desk during the test. Calculators are not allowed, either, but will not be needed. Ask me, and I will compute anything for you, if you need me to. Unless stated otherwise, please show all of your work and justify your answers in order to receive full credit.

For each problem you may use any results we discussed in class or stated in the text, except for the statement of the problem itself!

Good luck!

Date: October 7, 2018.

Problem 1. (1) Use the Gauss-Jordan method to determine the inverse

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}^{-1}.$$

(2) Find a permuted LDV factorization for

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}.$$

If you wish, you may use any computations that led to A^{-1} in Part (1).

Problem 2. *True or false*: The product AB of two singular $n \times n$ matrices A and B could be nonsingular. (Do not just answer this question. Back up your answer with a proof or a counterexample.)

Problem 3. Vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$ in $\mathbb{R}^4$ form the columns of a 4×4 matrix A:

$$A = \begin{pmatrix} | & | & | & | \\ \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{v}_4 \\ | & | & | & | \end{pmatrix}.$$

The row echelon form of A is

$$\begin{pmatrix} 3 & 1 & 7 & -1 \\ 0 & -4 & 8 & 2 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

(1) Do the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$ form a basis of $\mathbb{R}^4$? Justify your answer.

(2) Is $\mathbf{v}_3$ in the span $\operatorname{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4\}$? If not, explain why. If yes, write $\mathbf{v}_3$ as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_4$.

Problem 4. For the digraph 2.6.3(a) on p. 127 of the text,

(1) Find the incidence matrix;
(2) Find a basis of the cokernel of the incidence matrix.
(3) What is the dimension of the cokernel and what does it tell you about the number of independent circuits in the digraph.

Problem 5. Which of the following formulas define a norm on $\mathbb{R}^2$? Briefly justify your answer.

(1) $\|(x, y)\| = \min(|x|, |y|)$.

(2) $\|(x, y)\| = |x + y| + |x - y|$.

(3) $\|(x, y)\| = |x|^3 + |y|^3$.