Math 4242
Fall 2018
Sample Exam 2
11/26/2018
Time Limit: 50 minutes

Name (Print): ____________________

Instructor Alexander Voronov

The exam contains 7 pages (including this cover page) and 5 problems. Please check to see if any pages are missing.

- **You may not use** notes, books, the internet, etc. Only the exam paper, a pencil or pen may be kept on your desk during the test. Calculators are not allowed, either, but will not be needed. Ask me, and I will do any computation for you on my calculator.
- **You may use** any results we discussed in class or stated in the text, except for the statement of the problem itself!
- **Show all of your work and justify your answers**, unless stated otherwise, in order to receive full credit.
- **If you need more room**, use the back of the pages.
- **Please do not write in the table to the right**.

Problem	Points	Score
1	15	
2	10	
3	15	
4	15	
5	15	
Total:	70	

Good luck!

1. (15 points) In the following problem, use the standard Euclidean dot product.

   (a) (5 points) Find a basis for the orthogonal complement of $W \subset \mathbb{R}^4$ where

$$W = \operatorname{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 2 \\ 2 \end{pmatrix} \right\}.$$

   (b) (10 points) Find an orthonormal basis for the space $W \subset \mathbb{R}^4$.

2. (10 points) For the following problems, circle TRUE or FALSE. **If false, state why.**

   (a) (3 points) TRUE FALSE Let Q be an orthogonal matrix. Then $\det(Q) = \pm 1$.

   (b) (3 points) TRUE FALSE Every linearly independent basis is an orthogonal basis.

   (c) (4 points) TRUE FALSE The quadratic function

   $$f(x, y) = x^2 + 6xy - 2y^2 - 8x + 5y + 12$$

   has a unique global minimum.

3. (15 points) Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation whose matrix representation with respect to the standard basis is given by

$$A = \begin{pmatrix} 3 & 0 & 1 \\ 4 & -1 & 1 \\ -2 & 0 & 0 \end{pmatrix}.$$

Find the matrix representation of L with respect to the basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

4. (15 points) Describe a mass-spring chain with both ends fixed that gives rise to the potential energy function
$$3u_1^2 - 4u_1u_2 + 3u_2^2 + u_1 - 3u_2$$
and find its equilibrium configuration.

5. (15 points) Let A be a symmetric 3×3 matrix with eigenvalue and eigenvector pairs as follows:

$$\lambda_1 = 2, \qquad \mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},$$

$$\lambda_2 = -1, \qquad \mathbf{v}_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix},$$

$$\lambda_3 = -1, \qquad \mathbf{v}_3 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}.$$

(a) (3 points) What is the determinant of A?

(b) (3 points) Is A positive definite? Why or why not?

(c) (3 points) How many Jordan blocks for the Jordan canonical form of A have? Briefly justify your answer. (You do **not** need to find the Jordan canonical form to answer this question!)

(d) (6 points) Write out the spectral factorization of A if possible. If not, state why.