

**MATH 5615H: INTRODUCTION TO ANALYSIS I**  
**SAMPLE MIDTERM EXAM I**

You may not use notes, books, etc. Only the exam paper, a pencil or pen may be kept on your desk during the test. Calculators are not allowed, either, but will not be needed. Ask me, and I will compute anything for you, if you need me to. Unless stated otherwise, please show all of your work and justify your answers in order to receive full credit.

Good luck!

**Problem 1.** Construct a sequence whose set of cluster points is  $[0, 1]$ . Explain why it is exactly  $[0, 1]$ .

**Problem 2.** (1) Prove that  $||a| - |b|| \leq |a - b|$ .

(2) Let  $f$  be a function defined on a domain that includes a set  $E$  of real numbers and  $a$  be an accumulation point of  $E$ . Suppose  $\lim_{\substack{x \rightarrow a \\ x \in E}} f(x) = L \in \mathbb{R}$ . Prove that  $\lim_{\substack{x \rightarrow a \\ x \in E}} |f(x)| = |L|$ .

**Problem 3.** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a continuous, periodic function. Prove that its image  $f(\mathbb{R})$  is a closed and bounded interval.

**Problem 4.**

Let  $f(x)$  be a uniformly continuous function on an interval  $I$  and  $\{a_n\} \subset I$  is a Cauchy sequence. Show that  $\{f(a_n)\}$  is also Cauchy.

Why not check up all your work?

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