MATH 8211: COMMUTATIVE AND HOMOLOGICAL ALGEBRA PROBLEM SET 1 (PART II), DUE OCTOBER 1, 2003

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This is Part II of the first problem set.

I encourage you to cooperate with each other on the homeworks.

Reminder: all rings are commutative with an identity element 1, all ring homomorphisms carry 1 to 1, and a subring shares the same identity element with the ring.

Eisenbud: 1.24, 1.25, 2.22, 2.23, 2.25, 4.27 (Do not use Nullstellensatz, just generalize the argument we had in class for polynomials in two variables.)

Hint for **Fermat's Last Theorem**: **Just skip it**. I have realized it is much harder to prove (for $n = 3$ and 4) than I thought. For example, for $n = 3$, you need to consider two cases: when x, y, z are not divisible by 3 and when x and y are not divisible by 3, but z is. Then you need to show that the ring $\mathbb{Z}[\epsilon]$ for $\epsilon = \sqrt[3]{1}$ is Euclidean, for example, using the norm $N(\alpha)$ of $\alpha \in \mathbb{Z}[\epsilon]$ as an element of the extension field $\mathbb{Q}(\epsilon) \supset \mathbb{Q}$. Then, in the first case, $x + \epsilon y$ is a cube of an element of $\mathbb{Z}[\epsilon]$, up to a unit, while in the second case, $x + \epsilon y$ is $(1 - \epsilon)$ times a cube. Doing the same for $x - \epsilon z$, you may show you get a contradiction. See [Borevich, A. I. and Shafarevich, I. R. *Number theory.* Academic Press, 1966. Chapter III].

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