Expressions (4) permit us to write in effective form the two-zone solution of an algebraically integrable nonlinear wave equation.

One should note that the solution of the problem of effectivization of formulas of finite-zone integration is connected with deep problems of algebraic geometry. In the approach of [2] the solution of this problem, to which Novikov gave consideration, turned out to be equivalent to the solution of the Schottky problem on description of Jacobian τ-matrices [7-9]. Riemann, Frobenius, Netter, and others were concerned with generalization of Eq. (3) to the the case of nonhyperelliptic modules; in connection with this Weil formulated a problem which was recently solved by Igusa [10, 11]. A detailed account of the results mentioned here is given in [12].

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RELATIVE DISPOSITION OF THE SCHUBERT SUPERVARIETIES AND RESOLUTION OF THEIR SINGULARITIES

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Let G be a complex algebraic supergroup of one of the following types: $SL(m|n)$, $OSp(m|n)$, $\Pi Sp(m)$ and $Q(m)$, given together with its standard representation $T^{m/n}$. The group $OSp(m|n)$ is the automorphism group of a nondegenerate even symmetric bilinear form $b: T \tilde{\to} T^*$, $\Pi Sp(m)$ is that for an odd antisymmetric form $p: T \tilde{\to} T$ $(m = n)$. We call a G-invariant flag $0 \subsetneqq \mathcal{P}_1 \subsetneqq \ldots \subsetneqq \mathcal{P}_r = T$, where $r = m + n$ for all G, except $Q(m)$ when $r = m$, a complete G-flag.

In [1] a decomposition of the superspace of the complete G-flags into the Schubert supercells X_w, which are locally closed subsuperschemes and are numbered by the elements of the Weyl supergroup W, introduced there, is consructed. The relation $X_w \subset \overline{X}_w$, where $\overline{X}_w$ is the superscheme closure of X_w, gives an order in W. In the present note, we define this order in an intrinsic manner.

The Schubert supervarieties $\overline{X}_w$ are interesting also in that they provide natural examples of supervarieties with singularities. In the classicial geometry, results such as the proof of the rationality of singularities of the Schubert varieties, computation of the Chow ring of a flag space, the Demazure character formula can be obtained, relying on the construction of the Bott—Samelson scheme, giving a resolution of the singularities of a Schubert variety (see Bernshtein—Gel'fand—Gel'fand [2] and [3]). Here we propose a construction of a resolution of singularities of $\overline{X}_w$, following basicially the Bott—Samelson plan. In addition, our construction is essentially different from the classicial one, since not all Borel

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subgroups of G are conjugate, the group W does not have a system of representatives in G, and the root system of the supergroup G is, in general, not an abstract root system.

1. Flag Supervarieties. The sequence $I = (\delta_1, \ldots, \delta_r)$, where $\delta_i(I) = \dim \mathscr{S}_i - \dim \mathscr{S}_{i-1}$, is called the type of the complete flag $0 \subset \mathscr{S}_1 \subset \ldots \subset \mathscr{S}_r$. The supervariety of complete G-flags is, by definition, the disconnected union ${}^G F = \coprod {}^G F(I)$ of the connected (for $G \neq OSp(2r, 2s)$) spaces ${}^G F(I)$ of complete G-flags of type I (in the case $G = OSp(2r, 2s)$ each ${}^G F(I)$ consists of two connected components). The supervarieties ${}^G F(I)$ are orbits (in the supersense) of the supergroup G, and the decomposition of the flag space ${}^G F$ into connected components corresponds to the decomposition of the set of Borel subgroups of G into conjugacy classes.

2. Weyl Supergroups. Let ${}^G I$ be the set of all types of complete G-flags in T. Let us set ${}^{SL}W = S_{m+n}$ with the action on ${}^{SL}I$ by permutations, ${}^G W = \{w \in {}^{SL}W \mid w({}^G I) \subset {}^G I\}$ for G = OSp or ΠSp, and ${}^Q W = S_m$.

3. Superlength. For G = SL we define the basic reflections in the group ${}^G W$ as $\sigma_i = (i, i + 1)$; see [1] for remaining G. We define the superlength $\ell_{IJ}(w) = (\ell_0(w) \mid \ell_1(w)) \in Z \times Z$ of an element $w \in {}^G W$ for I, $J \in {}^G I$, and $w(I) = J$ as $\sum_{i=1}^{s} l_{I_{i-1}, I_i}(w_i)$, where $w = w_s \ldots w_1$ is the reduced decomposition of w in a product of basic reflections, $I_0 = I$, and $I_i = w_i(I_{i-1})$. In addition, in the case G = SL we have $\ell_{IJ}(\sigma_i) = 1|0$ if $J = \sigma_i(I) = I$, and $0|1$ if $J = \sigma_i(I) \neq I$ (for other G, $\ell_{IJ}(\sigma_i)$ are equal to $0|0$, $0|1$, $1|0$, or $1|1$; see [1] for the precise definitions).

4. Schubert Supercells. Let $(\mathscr{S}_i)$ be a tautological flag on ${}^G F$ and $\mathscr{S}_{ij} = p_1^* \mathscr{S}_i \cap p_2^* \mathscr{S}_j$ be the coherent sheaves on ${}^G F \times {}^G F$. The elements $Y_{w,IJ}$ of the flattening decomposition [4] of the space ${}^G F(I) \times {}^G F(J)$ for the system of sheaves $\{\mathscr{S}_{ij}\}$: $\coprod Y_{w,IJ} = {}^G F(I) \times {}^G F(J)$, where the union is taken over all $w \in W_{IJ} = \{w \in {}^G W \mid w(I) = J\}$, are called the Schubert supercells. The Schubert supercells have the following properties [1]: a) The projection p_1: $Y_{w,IJ} \to {}^G F(I)$ is surjective and $Y_{w,IJ}$ is an affine superspace of dimension $\ell_{IJ}(w)$ locally with respect to it. b) The restrictions of the sheaves $\mathscr{S}_{ij}$ to $Y_{w,IJ}$ are locally free; let us denote their ranks by $d_{ij,w}$. c) The supercell $Y_{w,IJ}$ is given uniquely by the matrix $(d_{ij,w})$. Let us define the Schubert supercells in ${}^G F(J)$ as $X_{w,IJ} = p_1^{-1}(x) \cap Y_{w,IJ} \subset p_1^{-1}(x) \cong {}^G F(J)$ for an arbitrary point $x \in ({}^G F(I))_{rd}$.

5. Order in a Weyl Supergroup. We call $w \in W$ a reflection if w is conjugate to a basic reflection in W.

Definition ($G \neq OSp(2r, 2s)$). (i) Let I, $J \in {}^G I$, w_1, $w_2 \in W_{IJ}$, and $\sigma \in W_{JJ}$ be a reflection. Then $w_1 \to w_2$ means that $\sigma w_1 = w_2$ and $\ell_0(w_2) = \ell_0(w_1) + 1$.

(ii) Let us set $w < w'$ if there exists a chain

$$w = w_1 \to w_2 \to \ldots \to w_k = w'.$$

Remark. Here and in the sequel, for $G = OSp(2r, 2s)$ in place of W_{IJ} it is necessary to consider each of the two orbits in W_{IJ} of a certain subgroup of W_{JJ} of index 2. This subgroup is the image of the Weyl group W^0 of the connected component of the identity of the group G_{rd} under the natural embedding in W_{IJ}. We point out that this subgroup coincides with W_{JJ} for the remaining G.

6. THEOREM. The following conditions are equivalent for I, $J \in {}^G I$ and w, $w' \in W_{IJ}$: a) $w' \leq w$. b) $d_{ij,w'} \geq d_{ij,w}$ for all i and j. c) $X_{w',IJ} \subset \overline{X_{w,IJ}}$. d) $(X_{w',IJ})_{rd} \subset \overline{(X_{w,IJ})_{rd}}$.

Remark. The equivalence a) $\Leftrightarrow$ d) shows that the order in W_{JJ} coincides with the standard order [2] in the Weyl group W^0 with respect to the embedding $W^0 \to W_{JJ}$ (see Sec. 5). The equivalence c) $\Leftrightarrow$ d) means that the decomposition into Schubert supercells is, in a natural sense, a supercellular complex.

7. Definition. Let X be a supervariety. A point $x \in X_{rd}$ is said to be nonsingular if there exists a neighborhood $U \subset X$ such that $x \in U$ and U is a nonsingular supervariety. The set of singular points of the supervariety X is conjugate to the set of nonsingular points in X_{rd}.

Since $X_{w,IJ}$ are nonsingular, the Schubert supervarieties $\overline{X_{w,IJ}}$ have singularities only at the points of $(\overline{X_{w,IJ}})_{rd} \setminus (X_{w,IJ})_{rd}$ and, a fortiori, have them there where the Schubert varieties $(\overline{X_{w,IJ}})_{rd}$ are singular.

8. The Bott–Samelson Superscheme. Let I, J $\in {}^G I$, $w \in W_{IJ}$, $w = s_k \ldots s_1$ be a reduced decomposition of w into basic reflections, and $w_i = s_i \ldots s_1$, $1 \le i \le k$. Let us define the sequence of projective superfibers $\mathrm{Spec}\, C = Z_0 \leftarrow Z_1 \leftarrow \ldots \leftarrow Z_k$ by the following inductive rule. (To economize space, we give the construction only for G = SL). The point Z_0 is the point $\{f_I\} \in {}^G F(I)_{rd}$, $f_I = (\mathscr{S}_1 \subset \ldots \subset \mathscr{S}_{m+n})$. Let Z_j be already constructed; for each superscheme S over C each S-point Z_j is a flag of type $I_j = w_j(I)$ in $T \otimes \mathscr{O}_S$. Let $s_j = (i, i+1)$. Then as Z_{j+1} we take the relative projective superspace $P_{Z_j}(\delta_i(I_{j+1}); \mathscr{S}_{i+1}/\mathscr{S}_{i-1})$ of relative dimension $\ell_{I_j, I_{j+1}}(s_j)$, where $(\mathscr{S}_i)$ is tautological flag on Z_j. By construction, all Z_j are nonsingular.

Definition. $Z_w = Z_k$ is called a Bott–Samelson superscheme (it clearly depends on the choice of the reduced decomposition of w).

9. THEOREM. There exists a morphism $Z_w \to {}^G F(J)$ that is a resolution of the singularities of $\overline{X_{w,IJ}} \subset {}^G F(J)$, i.e., a morphism $Z_w \to \overline{X_{w,IJ}}$ is defined, is surjective, and induces an isomorphism of the open subsupervariety in Z_w with $X_{w,IJ}$.

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SUPERAMENABILITY AND THE PROBLEM OF OCCURRENCE OF FREE SEMIGROUPS

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In connection with studies concerning the Hausdorff–Banach–Tarski paradox, von Neumann [1] considered the following problem. Let G be a group acting as a group of transformations on a set S and let $Z \subset S$ be a nonempty subset. The question is whether there exists a normed, on A, finitely additive G-invariant measure μ defined on the sigma-algebra of all subsets of S and assuming its values in the interval $[0, +\infty]$. Such measures are called invariant measures for the triple (G, S, A). Groups G for which invariant measures exist for the triple (G, G, G) were classified by von Neumann as a special class AG, the class of amenable groups. Theory and applications of amenable groups have been studied in the monograph [2].

Even when a group G is amenable, the problem of existence of an invariant measure for a triple (G, S, A) is not always decidable. In [3] Rosenblatt introduced the notion of a superamenable group: a group G is said to be superamenable if the problem of existence of an invariant measure is decidable for each triple (G, S, A), where A is not empty. Rosenblatt proved that a group of nonexponential growth is superamenable. He noted that if a group G contains a free subsemigroup L with two generators, then the triple (G, G, L) has no invariant measure. So a group possessing a free subsemigroup with two generators is not superamenable. Rosenblatt proved that a solvable group is superamenable if and only if it contains no free subsemigroup with two generators. In the same article he proposed the following conjecture ([3], p. 41; also see [9]):

A group is superamenable if and only if it is amenable and contains no free subsemigroup with two generators.

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