THEOREM. *Let* $X_{H,V}$ *be a stable summand of* $(BH_+)^\wedge_2$ *corresponding to the simple* $\mathbb{F}_2\,\mathrm{Out}(H)$*-module* V*, where* H *is a 2-group. The matrix*

$$\mathrm{rank}_{\mathbb{Z}_p} \mathrm{Hom}(X_{H,V}, X_{K,W})$$

is the Cartan matrix shown, where $\mathcal{X} = \mathrm{all}$ *and* $\mathcal{Y} = 1$.

THEOREM. *Over any field* R*, for any class of groups* $\mathcal{X}$ *closed under extensions and sections, the Cartan matrix* $C_R^{\mathcal{X},\mathcal{X}}$ *of globally defined Mackey functors* $\mathrm{Mack}_R^{\mathcal{X},\mathcal{X}}$ *is symmetric and non-singular.*

THEOREM. *Let* $(K, \mathcal{O}, k)$ *be a* p*-modular system. Then the Cartan matrices and decomposition matrices satisfy* $C_k = D^T C_K D$.

The table of values $S_{H,V}(G)$ of simple functors in $\mathrm{Mack}_{\mathbb{C}}^{1,1}$. Because the characteristic is zero we have $S_{H,V} = \Delta_{H,V}$. These are also the values of the simple functors in $\mathrm{Mack}_{\mathbb{C}}^{\mathrm{all},1}$.

Formal characters $\mathcal{X}$ = all or 1, $\mathcal{Y} = 1$, characteristic 0			$S_{H,V}$: 1	C_2	C_4		$C_2 \times C_2$			C_8				Q_8			D_8	
			1	1	1	-1	1	-1	2	1	-1_3	-1_5	-1_7	1	-1	2	1	-1
K, W	1	1	1															
	C_2	1	1	1														
	C_4	1	1	1	1													
		-1				1												
	$C_2 \times C_2$	1	1	1			1											
		-1						1										
		2		1					1									
	C_8	1	1	1	1					1								
		-1_3									1							
		-1_5				1						1						
		-1_7											1					
	Q_8	1	1	1	1									1				
		-1													1			
		2			1											1		
	D_8	1	1	2	1		1		1								1	
		-1		1			1		1									1

Table of values of simple functors in $\mathrm{Mack}_{\mathbb{F}_2}^{1,1}$.

Formal characters $\mathcal{X} = 1$, $\mathcal{Y} = 1$, characteristic 2			$S_{H,V}$: 1	C_2	C_4	$C_2 \times C_2$		C_8	Q_8		D_8
			1	1	1	1	2	1	1	2	1
K, W	1	1	1								
	C_2	1		1							
	C_4	1			1						
	$C_2 \times C_2$	1				1					
		2					1				
	C_8	1						1			
	Q_8	1							1		
		2								1	
	D_8	1					2				1

The Cartan matrix for $\mathrm{Mack}_{\mathbb{C}}^{1,1}$ in characteristic 0 is the identity matrix
Decomposition matrix for $\mathrm{Mack}_{\mathbb{C}}^{1,1}$ from characteristic 0 to characteristic 2:

Decomposition matrix $\mathcal{X}=1$, $\mathcal{Y}=1$ characteristic 2		$S_{H,V}$								
		1	C_2	C_4	$C_2\times C_2$		C_8	Q_8		D_8
		1	1	1	1	2	1	1	2	1
$S_{K,W}$ 1	1	1	1	1	1		1	1		1
C_2	1		1	1	1	1	1	1		1
C_4	1			1			1	1	1	1
	-1			1			1			
$C_2\times C_2$	1				1					2
	-1				1					
	2					1				
C_8	1						1			
	-1_3						1			
	-1_5						1			
	-1_7						1			
Q_8	1							1		
	-1							1		
	2								1	
D_8	1									1
	-1									1

Cartan matrix $C_{\mathbb{F}_2} = D^T D$ for $\mathrm{Mack}_{\mathbb{F}_2}^{1,1}$ in characteristic 2:

Cartan matrix $\mathcal{X}=1$, $\mathcal{Y}=1$ characteristic 2		$P_{H,V}$								
		1	C_2	C_4	$C_2\times C_2$		C_8	Q_8		D_8
		1	1	1	1	2	1	1	2	1
$S_{K,W}$ 1	1	1	1	1	1	0	1	1	0	1
C_2	1	1	2	2	2	1	2	2	0	2
C_4	1	1	2	4	2	1	4	3	1	3
$C_2\times C_2$	1	1	2	2	4	1	2	2	0	4
	2	0	1	1	1	2	1	1	0	1
C_8	1	1	2	4	2	1	8	3	1	3
Q_8	1	1	2	3	2	1	3	5	1	3
	2	0	0	1	0	0	1	1	2	1
D_8	1	1	2	3	4	1	3	3	1	9

Cartan matrix $C_{\mathbb{C}}$ for $\text{Mack}_{\mathbb{C}}^{\text{all},1}$ in characteristic 0:

Cartan matrix $\mathcal{X} = \text{all}$, $\mathcal{Y} = 1$ characteristic 0		$P_{H,V}$ 1	C_2	C_4		$C_2 \times C_2$			C_8				Q_8			D_8	
		1	1	1	-1	1	-1	2	1	-1_3	-1_5	-1_7	1	-1	2	1	-1
$S_{K,W}$	1 1	1															
	C_2 1	1	1														
	C_4 1	1	1	1													
	-1				1												
	$C_2 \times C_2$ 1	1	1			1											
	-1						1										
	2		1					1									
	C_8 1	1	1	1					1								
	-1_3									1							
	-1_5										1						
	-1_7											1					
	Q_8 1	1	1			1							1				
	-1						1							1			
	2		1					1							1		
	D_8 1	1	2			1		1								1	
	-1		1					1									1

Cartan matrix $C_{\mathbb{F}_2} = D^T C_{\mathbb{C}} D$ for $\text{Mack}_{\mathbb{F}_2}^{\text{all},1}$ in characteristic 2:

Cartan matrix $\mathcal{X} = \text{all}$, $\mathcal{Y} = 1$ characteristic 2		$P_{H,V}$ 1	C_2	C_4	$C_2 \times C_2$		C_8	Q_8		D_8
		1	1	1	1	2	1	1	2	1
$S_{K,W}$	1 1	1	0	0	0	0	0	0	0	0
	C_2 1	1	1	1	1	0	1	1	0	1
	C_4 1	2	2	4	2	0	4	3	1	3
	$C_2 \times C_2$ 1	2	2	2	4	0	2	2	0	4
	2	0	1	1	1	1	1	1	0	1
	C_8 1	3	3	6	3	0	10	5	2	5
	Q_8 1	3	3	4	5	0	4	6	1	6
	2	1	2	3	2	1	3	3	2	3
	D_8 1	5	7	8	10	2	8	8	1	16

Decomposition matrix for $\mathrm{Mack}_{\mathbb{C}}^{\mathrm{all},1}$ from characteristic 0 to characteristic 2:

Decomposition matrix		$S_{H,V}$								
$\mathcal{X}$ = all, $\mathcal{Y} = 1$ characteristic 2		1	C_2	C_4	$C_2 \times C_2$		C_8	Q_8		D_8
		1	1	1	1	2	1	1	2	1
$S_{K,W}$	1 1	1								
	C_2 1		1	1	1		1	1		1
	C_4 1			1			1	1	1	1
	-1			1			1			
	$C_2 \times C_2$ 1				1					2
	-1				1					
	2					1				
	C_8 1						1			
	-1_3						1			
	-1_5						1			
	-1_7						1			
	Q_8 1							1		
	-1							1		
	2								1	
	D_8 1									1
	-1									1

Table of values of simple functors in $\mathrm{Mack}_{\mathbb{F}_2}^{\mathrm{all},1}$:

Formal characters			$S_{H,V}$								
$\mathcal{X} = \mathrm{all}$, $\mathcal{Y} = 1$			1	C_2	C_4	$C_2 \times C_2$		C_8	Q_8		D_8
characteristic 2			1	1	1	1	2	1	1	2	1
K, W	1	1	1								
	C_2	1	1	1							
	C_4	1	1		1						
	$C_2 \times C_2$	1	1			1					
		2		1			1				
	C_8	1	1					1			
	Q_8	1	1						1		
		2								1	
	D_8	1	1	2			2				1