Math 5345H Topology
Chapter 0

For sets X,Y we define

Y $\subseteq$ X means $y \in Y \Rightarrow y \in X$, every element of Y is a member of X

Y $\subset$ X means $y \in Y \Rightarrow y \in X$ and $Y \neq X$

X-Y or $X \setminus Y = \{x \in X \mid x \notin Y\}$ = elements of X not in Y

The empty set $\emptyset$

Notation to define a set, such as

X x Y = $\{(x,y) \mid x \in X, y \in Y\} = \{(x,y) : x \in X, y \in \}$

A function f : X -> Y is a rule that, given $x \in X$, assigns an element $f(x) \in Y$.
We may write just f

The identity function 1_X : X -> X is $1_X(x) = (x)$ for all $\forall x \in X$

The image f(X)
$= \{y \in Y \mid \exists x \in X, y = f(x)\} = \mathrm{Im}(f)$
there exists

For subsets W, W′ of X the union and intersection

$W \cup W' = \{x \in X \mid x \in W \text{ or } x \in W'\}$

$W \cap W' = \{x \in X \mid x \in W \text{ and } x \in W'\}$

If Z is a subset of Y, the inverse image of Z (or preimage) is

$f^{-1}(Z) = \{x \in X \mid f(x) \in Z\}$

A function f is one-to-one or injective if

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \; \forall x_1, x_2 \in X$

A function f is onto or surjective if

$\forall y \in Y, \exists x \in X, f(x) = y$

Both injective and surjective is called bijective and there is an inverse function.

$f : X \to Y$ gives a function $f^{-1} : Y \to X$

If f : X -> Y and g : Y -> Z are functions, the composite function $g \circ f = gf : X \to Z$

defined by $(gf)(x) := g(f(x))$
definition of left side

A relation on a set X is a subset of $X \times X$.
It is a set of pairs (a,b), $a, b \in X$.
The book writes the subset as $\sim$.
We write $a \sim b$ to mean (a,b) lies in the subset.
Example: $X = \{1, 2\}$.
$\{(1,2)\}$ is a relation. $1 \sim 2$ is correct.

A relation is called an equivalence relation if it satisfies
(r) $x \sim x$ $\forall x \in X$ reflexivity
(s) $x \sim y \iff y \sim x$ symmetry
(t) $x \sim y$ and $y \sim z \Rightarrow x \sim z$. transitivity.

How many of (r), (s), (t) does the example satisfy?
0
1
2
3

Proposition. If $\sim$ is an equivalence relation on X then each element of X belongs to precisely on equivalence class.
It is the same as expressing X as a disjoint union of subsets.
Definition. The equivalence class of $x \in X$ is $[x] = \bar{x} = \underline{x} = \{ y \in X \mid x \sim y \}$

Proof. If $y \in [x]$ then $x \in [y]$ and $[x] = [y]$. To see this if $u \in [x]$ then $x \sim u$ and $x \sim y$. Now $y \sim x$ and $y \sim x \sim u$ so $y \sim u$, $u \in [y]$
Thus $[x] \subseteq [y]$. By symmetry $[y] \subseteq [x]$ so $[x] = [y]$. Two equivalence class are the same or disjoint. Every element is in an equiv. class.

We know
$\mathbb{R}$ $\mathbb{C}$ $\mathbb{Z}$ $\mathbb{N}$ $\mathbb{Q}$
$[a, b) = \{ x \in \mathbb{R} \mid a \leq x < b \}$